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**GAS DISTRIBUTION  
IN SHALLOW LARGE  
DIAMETER PACKED  
BEDS**

A THESIS SUBMITTED FOR THE DEGREE  
OF DOCTOR OF PHILOSOPHY

BY

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DECEMBER 1984



*In the Name of Allāh,  
the Most Compassionate, the Merciful*

And we want to bestow our favours on  
those who were oppressed on earth, and  
make them leaders of mankind and make  
them the inheritors. (*The Story*, 28:5)

**TO AL-MAHDI (P.B.U.H.)**

The work described in this thesis was carried out between 1981 and 1984 in the Department of Chemical Engineering, University of Aston in Birmingham. It has been carried out independently and has not been submitted for any other degree.

A handwritten signature in black ink, consisting of a long horizontal line followed by a stylized, looped flourish.

Quasim H. Ali

THE UNIVERSITY OF ASTON IN BIRMINGHAM  
GAS DISTRIBUTION IN SHALLOW LARGE DIAMETER PACKED BEDS

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A thesis submitted for the degree of  
Doctor of Philosophy, December 1984.

SUMMARY

It is important to maintain a uniform distribution of gas and liquid in large diameter packed columns to maintain mass transfer efficiency on scaling up.

This work presents measurements and methods of evaluating maldistributed gas flow in packed columns. Little or no previous work has been done in this field.

A gas maldistribution number,  $\phi$ , was defined, based on point to point velocity variations in the gas emerging from the top of packed beds.  $\phi$  has a minimum value for a uniformly distributed flow and much larger values for maldistributed flows.

A method of testing the quality of vapour distributors is proposed, based on the variation of  $\phi$  with packed height. A good gas distributor requires a short packed depth to give a good gas distribution.

Measurements of gas maldistribution have shown that the principle of dynamic similarity is satisfied if two geometrically similar beds are operated at the same Reynold's number.

The validity of  $\phi$  as a good measure of gas maldistribution, and the principle of dynamic similarity are tested statistically by Multi-Factor Analysis of the variance, and visually by the response surfaces technique.

Pressure distribution has been measured in a model of a large diameter packed bed, and shown to be associated with the velocity of the gas in a tangential feed pipe. Two simplified theoretical models are proposed to describe the flow of gases through packed beds and to support the principle of dynamic similarity. These models explain why the packed bed itself causes the flow of gas to become more uniformly distributed.

A 1.2m. diameter scaled-down model was constructed geometrically similar to a 7.3m. diameter vacuum crude distillation column. The previously known internal cylinder gas distributor was tested. Three new distributors suitable for use in a large diameter column were developed and tested, these are:

- a) Internal Cylinder with Slots and Cross Baffles
- b) Internal Cylinder with Guides in the Annulus
- c) Internal Cylinder with Internal Cross Baffles

It has been shown that (c) is an excellent distributor.

PUMP AROUND. GAS DISTRIBUTORS. GAS MALDISTRIBUTION. FLUID MECHANICS. SCALE-UP.

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SECTION I  
INTRODUCTION

The use of packed towers of large diameters for distillation and absorption services has seen a steady increase in recent years. This has been mainly due to the lower pressure drops obtained in packed towers, compared to trayed towers for similar duties when implementing energy saving schemes.

A serious problem arises in the use of large diameter packed beds, namely, the possibility of an uneven distribution of gas and liquid through the packing. A non-uniform distribution of gases and liquids has a much greater effect on the performance of large diameter columns than on small diameter columns. For maximum efficiency, both the gas and liquid should be uniformly distributed over the bed cross-section, and this uniformity should persist through the packing height.

Although a considerable amount of work has been done on liquid distribution in packed columns, little or nothing has been done on gas distribution. Gas distribution is rarely a problem for small diameter columns where the packed height is very much larger than the diameter of the column. Now that larger diameters are being used the problem of gas distribution has become more important. The problem is particularly important when the gas or vapour inlet pipe is small compared with the diameter of the column. This can occur in revamping situations, but it is also found in large new plants where there are economic advantages from limiting the size of the feed pipe. A particularly difficult example is the pump around section in a vacuum crude tower, where the bed is 7.3m in diameter packed to a depth of 1 to 2m, and with a pressure drop of 1 mm Hg. across the

packing, and a vapour inlet pipe of 1m in diameter. This type of application has revealed a great need for better understanding of the problems of achieving a uniform gas distribution in packed beds of large diameter, and often with a diameter to height ratio of greater than one.

Although real packed columns usually involve the contact of gas and liquid, an investigation into maldistribution based on gas flow alone is a good starting point. The experimental work which will be discussed fully here, involves a detailed study of gas maldistribution and the use of laboratory models to investigate the gas flow patterns of full scale columns. Work was performed on a small scale model and on a prototype which are exactly geometrically and dynamically similar. The condition of dynamical similarity between two cases with geometrically similar boundaries is found to be satisfied if the Reynolds number has the same value in each case. Although it is to be expected that for compressible gas flow, the Reynolds number is not enough to define dynamic similarity, it was found to be sufficient for air flow through packed beds where the pressure drops and velocities are small.

However, Porter<sup>(25)</sup> (1982) indicates that the use of the principle of dynamic similarity to study gas distribution in a packed bed needs a concept to deal with the lack of geometric similarity resulting from the randomness of the bed.

In this work a maldistribution factor, " $\phi$ ", which is related to the standard deviation,  $\sigma^2$ , is defined in terms of the point to point velocity variations of the flow of gas emerging from the top of packed beds. The effectiveness of  $\phi$  for describing gas maldistribution in packed beds, the principle of dynamic similarity, and scale-up rules, are studied statistically by Multi-Factor Analysis



of the Variance, which shows that  $\phi$  is a satisfactory measure of flow uniformity, and that the gas flow patterns in two geometrically and dynamically similar packed beds of different size are similar. As a result of the randomness of the packed beds it is found that  $\phi$  never reaches a zero value, however good the distribution. Rather, it approaches a minimum value,  $\phi_{\min}$ , which represents the best possible gas distribution it is possible to achieve. It is demonstrated that the packed bed itself acts as a distributor and that at a sufficiently large depth of packing, gas distribution becomes as good as possible and  $\phi$  reduces to  $\phi_{\min}$ .

The fluid mechanics of maldistributed gas flow in packed beds is then studied, and experimental measurements are made on the pressure distribution in the bed. Following this, two simple theoretical models are proposed in this work to describe these flow patterns. The understanding of the fluid mechanics gained in this work permits the development of new improved low pressure drop gas distributors. These are:

- (1) Internal Cylinder with Four Slots and Cross Baffles.
- (2) Internal Cylinder with Guides in the Annulus.
- (3) Internal Cylinder with Internal Cross Baffles.

The reliability of  $\phi$  in describing gas maldistribution in packed beds permits the establishment of a method to test the quality of gas distributors based on the variation of  $\phi$  with packed depth. A good gas distributor is that one in which only a small packed depth is required for  $\phi$  to reach  $\phi_{\min}$ .

The establishment of the scale-up rules enables the construction of a laboratory size model of the full scale pump around section mentioned above, and this was used to test the new distributors. The Internal Cylinder with Internal Cross Baffles is found to be the best.

SECTION II  
LITERATURE SURVEY

2.1      The Effect of Maldistribution on the Performance of  
Packed Beds

It is known that poor liquid and gas distribution has an adverse effect on the performance of packed columns. For maximum efficiency both the liquid and gas should be uniformly distributed over the bed cross-section, and this uniformity should persist throughout the packed height.

The effect of maldistribution can be shown diagrammatically on the McCabe Thiele diagram where different parts of the packed bed have different liquid/gas ratios and a "pinch" can occur in some sections of the packed bed, see Fig. 2.1.

Some workers have studied the wall effect on the maldistribution of gas in dry or wetted beds, and more workers have studied the maldistribution of liquid in packed beds where the liquid is the only stream flowing through the bed.

Indeed, a few workers<sup>(1, 2)</sup> have attempted to study liquid maldistribution in the presence of a counteracting gas stream, but none of them have included the effect of the gas maldistribution on the mass transfer performance of the packed bed. All the previous workers on the effect of liquid maldistribution on mass transfer have assumed a uniform gas flow through the bed and the problem was defined as a case of liquid maldistribution only.

The first model for the effect of maldistribution on mass transfer put by Mullin<sup>(1)</sup> (1957) is a very much simplified one. The column was assumed to be divided longitudinally into two equal sections by an impermeable membrane as shown in Fig. 2.2.

The two sections take an equal vapour flow but unequal



liquid rates. It was assumed that there is no mixing of gas or liquid across this theoretical membrane, and that within each section, the distribution of gas and liquid is uniform. A degree of maldistribution,  $M$ , was defined as the ratio of the liquid rate to gas rate, in each section.

Maldistribution  $M = 1$  means that the liquid rate is the same in each theoretical section;  $M = 2$  means that one section takes twice as much liquid as the other.

$G, G_1$  and  $G_2$  = The overall, Section 1, and Section 2 gas mass velocities respectively and

$$G = G_1 + G_2 \quad \dots (2.1)$$

$L, L_1$  and  $L_2$  = The overall, Section 1, and Section 2 liquid mass velocities respectively, and

$$L = L_1 + ML_1 \quad \dots (2.2)$$

$$R_{\text{overall}} = \frac{G}{L}; \text{ the overall gas:liquid ratio } \dots (2.3)$$

$R_{\text{mean}}$  = The arithmetical mean of the ratios existing in each of the two hypothetical sections.

$$\text{i.e. } R_{\text{mean}} = \frac{1}{2} \left( \frac{G_1}{L_1} + \frac{G_2}{L_2} \right) \quad \dots (2.4)$$

Then

$$R_{\text{mean}} = R_{\text{overall}} \cdot \frac{(M+1)^2}{4M} \quad \dots (2.5)$$

If  $m$  = The slope of the equilibrium line

$L_m, G_m$  = The molar liquid and gas flow rates respectively

Then equation (2.5) will be as follows:

$$m \left( \frac{G_m}{L_m} \right)_{\text{mean}} = \frac{(M+1)^2}{4M} \cdot m \left( \frac{G_m}{L_m} \right)_{\text{overall}} \quad \dots (2.6)$$

Therefore, liquid maldistribution increases the value of the operating gas:liquid ratio  $G_m/L_m$ , i.e., it decreases the slope of the operating line  $L_m/G_m$ , resulting in a closer approach of this line to the equilibrium line (pinch), at one end of the tower.

The effect of  $M$  on the number of transfer units  $N_{OG}$  is shown in Fig. 2.3.

Mullin's method is only useful in explaining the observed reduction in the performance below predicted theoretically. It does not explain the reduction in the efficiency with increasing the diameter of the bed.

Huber and Hiltbrunner<sup>(2)</sup> (1966) have introduced the idea of cross mixing of the vapour or liquid which acts to cancel out the effect of maldistribution of the liquid. Their theory suggests that the number of transfer units of packed columns depends on the degree of maldistribution and on the amount of cross mixing,  $\tau_r$ , produced in the gas and liquid streams by the movement around the packing element, see Fig. 2.4.

$$\text{Where } \tau_r = -D \frac{\partial y}{\partial r} \quad \dots\dots (2.7)$$

$\tau_r$  = The amount of cross mixing or the horizontal mass transfer.

D = The diffusion constant.

y = The mole fraction in gas phase.

r = Radius of the packed bed.

Huber and Hiltbrunner predicted that in a small diameter column, only a small amount of cross mixing is required to put right a large amount of maldistribution, but that in a large diameter column, (more than 0.5m), the cross mixing will be less effective. Thus, large diameter packed beds are more likely to suffer from maldistribution than small diameter beds. That is, the performance of large diameter packed beds depends greatly on designing effective liquid and gas distributors. If the liquid and gas are uniformly distributed, column scale-up is satisfactory; if not, large diameter columns will fail.

## 2.2 Design of Distributors

Most packed tower failures are associated with maldistribution

of the vapour or liquid. The failures of "standard" packed towers is mainly due to liquid maldistribution, because in such towers the ratios of packed height to column diameter and vapour feed inlet diameter to column diameter are large. This causes the gas distribution to be usually satisfactory. Thus, there is as yet no scientific basis for the design of gas distributors.

A large diameter vacuum crude fractionation tower is chosen as an example of an extreme need of liquid and gas distribution devices.

#### 2.2.1 Description of the Vacuum Crude Fractionation Column

Vacuum crude fractionation towers work at a very low pressure to increase the relative volatility of the light components in the crude. In the past, when trays were used it was found that to maintain low partial pressure of the vapour, it was necessary to provide "Open" steam to compensate for the pressure drop through the trays.

Modern vacuum crude fractionators make the use of the low pressure drop of packed beds and in most cases use of open steam is no longer necessary.

The example used in this work is a pump around vacuum crude fractionation tower 7.3m in diameter, which consists of three pump around sections, see Fig. 2.5: a bottom stripping section; a heavy gas oil section; and a light gas oil section. Each pump around section contains a bed, which is usually packed to a depth of no more than 2m. The pump around packed bed acts as a condenser. That is, some of the rising vapour is condensed by the liquid which is recycled round the bed passing through an external heat exchanger. The use of the pump around condensing systems causes some of the heat of

condensation to be removed at a high temperature (higher than that of the conventional condensation at the top of the column). This permits more heat recovery to be achieved by exchange of heat to the crude oil feed which is used to cool the pump around heat exchangers.

The section of the vacuum crude fractionator under investigation in this work is the stripping pump around section which is immediately above the feed inlet. Most of the liquid in the feed, which is a mixture of vapour and liquid, is flashed in this section due to the reduced pressure, leaving the slop wax flowing down the column to be removed at the bottom.

The rising vapour passes through a draw-off tray, (troughs and slots type), such that the rising vapour passes through the slots to the packed bed above, and the falling liquid is collected by the troughs and drawn out of the column to a heat exchanger for heat recovery and cooling of the required product which is separated, and the rest of the liquid is recycled to the column above the bed. This liquid will contact the rising vapour in the bed and the condensation process takes place. The remaining vapour goes up for further separation.

There are many practical problems associated with these large diameter columns operated under a reduced pressure, some of which are listed below.

(1) Vapour phase maldistribution is very severe in the packed bed due to the highly maldistributed vapour feed which enters the bottom of the bed and the short depth of the bed compared to its diameter. These two factors reduce the efficiency of the bed due to poor contact between the gas and liquid.

(2) The size of the inlet nozzle is determined by the

flow rate and the degree of vaporization in the feed area. The conventional distillation columns with natural circulation reboilers necessarily have relatively large area inlets, because a large diameter pipe is required to minimize pressure difference between the top of the reboiler tubes and the top space in the column. The vacuum crude fractionator is fed by vapour (and liquid) from the bottom of the atmosphere pressure column which is passed through a pipe still to gain heat. Thus, a large diameter feed inlet is not required. Smaller pipe diameters (about 1m) are used to reduce the cost of the pipe and column.

(3) The pressure drop through these columns is very important in the process of fractionation. A lower pressure drop means a cheaper operating cost by avoiding steam distillation. This necessity limits the design of any gas distributor. Gas distributors (and all the column internals) should be designed to produce the lowest possible pressure drop.

(4) Due to the large diameter, the column internals, such as the draw-off tray, the support plate, etc., need to be supported by a mechanically strong support beam, which increases the capital cost and the complexity of the tower.

### 2.2.2 Liquid Distributors

In vacuum crude fractionators, spray type liquid distributors are usually used due to low liquid rates involved in such towers.

Liquid distributors are very important for the successful operation of a packed bed. An ideal liquid distributor should have the following attributes:

- 1 - Uniform-liquid distribution.
- 2 - Resistance to plugging and fouling.



3 - High turndown ratio.

4 - High free area for gas-flow.

5 - Sectional construction for installation through manways.

Two basic types of distributor are commonly used: the orifice type and the weir type. Both are gravity flow devices.

Orifice type devices are available in two basic designs: pan or ladder type. The ladder type (Fig. 2.6), is made from perforated pipes. It has a large free area for gas flow. In small diameter towers it is often more convenient to use a pan type orifice distribution.

In weir type distributors, (see Fig. 2.7), a series of closed end troughs are placed across the tower. Notches are cut in the sides of the troughs. The incoming liquid is distributed to the troughs in the appropriate amounts by closed end troughs which rest upon them.

### 2.2.3 Gas Distributors

Gas distributors are most important in large diameter towers packed to short depths, as is the case of the pump around beds. A good gas distributor should have the following features:

1 - Uniform-gas distribution.

2 - Low pressure drop.

3 - Sectional construction.

Two types of gas distributor are commonly used according to the way of introducing the feed. For a tangential feed inlet, an internal cylinder type gas distributor is commonly used. The internal cylinder is built inside the column below the bed, such that it leaves an annular space for the swirling gas to flow. The tangential motion of the feed helps in separating the entrained

liquid from the vapour. It is believed that the annular gap, (see Fig. 2.8) causes the vapour to be uniformly distributed around the base of the internal cylinder.

In the radial feed inlet situations, the feed inlet is extended inside the column, (see Fig. 2.9). Holes are cut in the sides of the extension which is closed at its end. The vapour is caused to leave the extended feed inlet through these holes.

## 2.3 Theories of Distribution in Packed Beds

Very little work has been done on the distribution of gases through packed beds, but a considerable investigation of liquid distribution is found in the literature and many theories have been developed to explain the nature of liquid distribution in packed beds.

### 2.3.1 Theories of Liquid Distribution

The distribution of liquids through randomly packed beds has been studied by many workers<sup>(3-16)</sup>.

There are different kinds of models proposed in the literature to describe the spread of liquid in the packing. In fact, these models can be specified into the following types:

Random walk and diffusion models.

#### (a) Random Walk Models

At very early stages, Thormann<sup>(3)</sup> (1928) suggested that the spread of liquids in packing may be described as one of a random walk of particles of liquid over the surface of the packed bed. Scott<sup>(4)</sup> (1935) and Tour and Lerman<sup>(5,6)</sup> (1939 and 1943) proposed a statistical model based on the random walk probability theory in the absence of wall effects. This work showed an expected normal distribution of liquids below a point source. However, the

mathematical complexities of the random walk theory are such that it is limited to a single point or line source and it is not suitable to predict, for example, the flow below a distributor or the wall effect.

(b) Diffusion Models

Because of this disability in random walk models and because the mathematics of diffusion are in general more convenient than those of random walk, Cihla and Schmidt<sup>(7)</sup> (1957), Porter and Jones<sup>(8)</sup> (1963), Dutkai and Ruchenstein<sup>(9)</sup> (1968) and Onda and Takeuchi<sup>(10)</sup> (1973) used a diffusion type equation. A radial spread factor is applied in this equation by which the spreading of a liquid stream in a packed bed is characterised. Porter and Jones<sup>(8)</sup> developed a theoretical model to predict the liquid distribution in terms of two factors, a liquid spread factor and a wall factor. They carried a mass balance on an elemental volume  $dx dy dz$ , shown in Fig. 2.10, which results in:

$$D \left[ \frac{\partial^2 f(x,y,z)}{\partial x^2} + \frac{\partial^2 f(x,y,z)}{\partial y^2} \right] = \frac{\partial f(x,y,z)}{\partial z} \quad \dots (2.8)$$

where

$D$  = liquid spread factor (dimensions:L).

$f(x,y,z)$  = Vertical flow of liquid per unit area

in unit time expressed as a function of  $x$ ,  $y$  and  $z$ .

$x,y,z$  = Spatial co-ordinates.

This theoretical model predicts flow profiles in a packed column but at small packed depths, the model overestimates the flow next to the wall.



(c) Other Models

There are many other models proposed in this field. Le Goff and Lespinasse<sup>(11)</sup> (1962) suggested according to their observations that the liquid flow pathways are stable with respect to time. Thus the liquid must follow "Preferred Paths" in the bed, and the liquid cannot be considered to take a random movement as it descends through the packing.

Porter<sup>(12)</sup> (1968) accepted the stability of the liquid flow pattern but proposed that if the stable flow paths change direction in a random manner then diffusion theory may still be used to predict the expected flow distribution. The agreement between experiment and the predicted flow per unit area will then depend on the number of flow paths, or "Rivulets", collected in the experiment. If a large number of rivulets are collected the agreement will be better than when a small number of rivulets are collected.

Porter, Barnett and Templeman<sup>(13)</sup> (1968) then used a statistical method based on  $\chi^2$  goodness of fit test to calculate the number of the rivulets collected in an experiment from the agreement (or lack of agreement) between experiment and theory.

A model proposed by Charpentier and Le Goff<sup>(14,15)</sup> (1968) consists of representation of the liquid flow in the form of "Films, Rivulets, and Drops", which describes the form of configuration and texture of the liquid distribution.

Bemer and Zuiderweg<sup>(16)</sup> (1978) concluded that the bulk of the liquid is flowing through "Stable Channels" through the bed even at good wetting conditions and that a large proportion of the axial mixing has to be attributed to the irregular flow distribution.

There are more recent models, such as the "Parallel Channels Model" and the "Triregional Model" and many others which are

related to the dynamic behaviour of liquid flow and channelling through packed beds.

### 2.3.2 Point to Point Variation in Liquid Flow

It is obvious from the experimental work performed by the above workers that the liquid leaves the bottom of the bed non-uniformly, and that there is a point to point variation in liquid flow through the packing. Porter<sup>(12)</sup> described this phenomenon in his theoretical model, "The Rivulet Model", and suggested that the liquid flows in the form of rivulets which sometimes coalesce to form larger rivulets, and sometimes break up into smaller ones.

Similar point to point variations in gas flow through packed beds are dealt with in this work.

### 2.3.3 Gas Distribution

Nothing has been done on maldistribution of gas flow in packed beds except the work performed by Zaytoun<sup>(17)</sup> (1981), which describes the gas flow above a randomly packed bed, and which forms the starting point for this work. A hot-point anemometer was used to measure air velocity at several points above a 38 cm packed bed over four equally spaced diameters. The bed was randomly packed with 1.6 cm metal Intalox packing to depths of 12.7 and 25.4 cm. A maldistribution factor  $\phi$  was defined as follows:

$$\phi = \sum \left(1 - \frac{v}{\bar{v}}\right)^2 \quad \dots\dots (2.9)$$

where  $v, \bar{v}$  = Point velocity and average velocity respectively  
(measured in ft/min).

The lowest possible value of this maldistribution factor,  $\phi_{\min}$ , was obtained and further experiments were performed to investigate the effects of bed height and inlet diameter on the maldistribution factor, see Fig. 2.11. The work showed point to point velocity variations even in deep beds and that the gas distribution is improved with packed height, that is the packed bed itself is a gas distributor. Zaytoun's work<sup>(17)</sup> provides a starting point to this work.

All the other experimental work on gas distribution which has been done, has been concerned with the variation of gas flow near the column wall and the voidage variation across the diameter of packed beds.

Saunders and Ford<sup>(18)</sup> (1940) measured the velocities of a gas emerging from the top of a bed packed with uniform spheres. Their results showed a uniform gas velocity distribution in the pack, and about 50% higher velocities in the annular region around the wall, approximately one packing diameter wide.

Schwartz and Smith<sup>(19)</sup> (1953) have predicted that 30 to 100% higher velocity in the wall region than the average velocity in the centre of the packed bed. They indicated that an important velocity maldistributions exist across the packed bed unless the column to packing diameter ratio is greater than 30.

These workers and many others have performed their works on columns with a large packed height to bed diameter ratios, and all agree that higher gas velocities exist near the column wall than in the rest of the bed, like that obtained by Calderbank and Pogorski<sup>(20)</sup> (1957), shown in Fig. 2.12. Such a velocity distribution is due to the voidage distribution in packed beds.

Roblee et al., Baird and Tierney<sup>(21)</sup> (1958) found a decrease in the void fraction from a value of unity near the wall which oscillates in a damped fashion up to a distance of 5 particle diameters. Afterwards the porosity is effectively constant as shown in Fig. 2.13. Such nonuniformities cause channelling which causes a poor performance of packed beds.

#### 2.4 Theoretical Analysis of Flow through Packed Beds

It was seen that a very little has been done on the investigation of gas maldistribution in packed beds. However, a few workers<sup>(22-25)</sup> have attempted to analyse, theoretically, the flow of fluids through packed beds. These theoretical studies involve the following categories: Dimensional analysis, Dynamic similarity and Gas maldistribution number.

##### 2.4.1 Dimensional Analysis of Two-Phase Flow

Johnston and Thring<sup>(22)</sup> (1957) demonstrated the incompatibility of choosing values of dimensionless groups when considering the simultaneous effect of the liquid and gas streams on geometrically and dynamically similar packed beds of a different size. In gas-liquid packed towers the flow pattern of the gas phase is controlled by the viscosity of the gas and the pressure difference; the flow of liquid is controlled by liquid viscosity and the force of gravity. The criterion for liquid flow in a packed tower, according to Johnston and Thring, was derived from Nusselt's equation for the isothermal streamline flow of liquid down a vertical wall

$$\frac{W}{l} = \frac{\rho_1^2 g \delta^3}{3\mu_1} \quad \dots\dots (2.10)$$

where,  $W$  = liquid rate, kg/sec.

$l$  = length of wetted perimeter, m .

$\delta$  = thickness of streamline film, m .

$\rho_1, \mu_1$  = density, viscosity of liquid at film temperature  
in kg.m.sec. units .

$g$  = acceleration of gravity, m/sec<sup>2</sup> .

For a complete tower packing, this equation may be written

$$Ld = \frac{1}{C} \frac{\rho_1^2 g \delta^3}{\mu_1} \quad \dots (2.11)$$

where,  $L$  = Superficial mass velocity of the liquid, kg/m<sup>2</sup>.sec.

$d$  = Diameter or equivalent diameter of packing elements, m.

$C$  = A constant .

which gives the dimensionless equation

$$\frac{\delta^3}{d^3} = C \frac{L\mu_1}{\rho_1^2 d^2 g} \quad \dots (2.12)$$

For kinematic similarity,  $\delta/d$  is constant, whence

$$\frac{L\mu_1}{\rho_1^2 d^2 g} = \text{constant} \quad \dots (2.13)$$

This is the similarity criterion for the liquid phase.

For the gas phase, the similarity requirement is a constant

Reynolds number:

$$\frac{Gd}{\mu_g} = \text{constant} \quad \dots (2.14)$$

where,  $G$  = Superficial mass velocity of the gas, kg/m<sup>2</sup>.sec.

$\mu_g$  = Viscosity of the gas, kg/sec.m.



For chemical similarity, as defined by Johnston and Thring, there is a further requirement of constant liquid/gas rate;

$$\frac{L}{G} = \text{constant} \quad \dots\dots (2.15)$$

In homologous systems, these three requirements reduce to

$$\frac{L}{d^2} = \text{constant} \quad \dots\dots (2.13a)$$

$$Gd = \text{constant} \quad \dots\dots (2.14a)$$

$$\frac{L}{G} = \text{constant} \quad \dots\dots (2.15a)$$

which are incompatible when  $d$  is varied.

Therefore, packed towers with two phase flow cannot be scaled up due to the incompatibility problem.

But, it is of value to know the requirements for establishing the rules of scale-up of packed beds. Thus it is possible to investigate flow patterns using models with the gas phase only.

#### 2.4.2 Dynamic and Geometric Similarity in Single-Flow Packed Beds

Moor<sup>(23)</sup> (1964), has suggested that two steady flows, at high speeds compared with the speed of sound, are dynamically similar if the Mach number,  $M$ , Reynolds number,  $Re$ , as well as many other properties of the fluid are similar. These include the equation of state (for relating changes of density to changes of pressure and temperature), specific-heat (for relating changes of temperature to changes of kinematic energy), and results on thermal conductivity and on the variation of viscosity and thermal conductivity with

temperature. The following is then a list of nondimensional parameters used in dynamic and geometric similarity:

$$M = \frac{V}{A} \quad \text{..... (2.16a)}$$

$$Re = \frac{\rho V d}{\mu} \quad \text{..... (2.16b)}$$

$$Pr = \frac{\mu C_p}{k} \quad \text{..... (2.16c)}$$

$$\gamma = \frac{C_p}{C_v} \quad \text{..... (2.16d)}$$

where

$V$  = Velocity of the fluid, (m/sec.) .

$A$  = Velocity of the sound in the air, (m/sec) .

$\rho, \mu$  = Density, ( $\text{kg/m}^3$ ) and viscosity, ( $\text{kg/m} \cdot \text{sec.}$ ) respectively .

$C_p, C_v$  = Specific heat at constant pressure and constant volume respectively, ( $\text{kg-cal/kg} \cdot ^\circ\text{C}$ ) .

$k$  = Thermal conductivity, ( $\text{kg-cal/m} \cdot \text{sec} \cdot ^\circ\text{C}$ ) .

$Pr$  = Prandtl Number, (dimensionless) .

$\gamma$  = Ratio of specific heats, (dimensionless) .

It is in general assumed that the Prandtl number depends on the nature of the fluid only. Also, it is often assumed that the ratio of the specific heats  $\gamma$  does not vary with temperature, pressure etc. Therefore, if the same fluid is used in the model and the prototype (as is the case in this work), then only  $M$  and  $Re$  should be kept constant, but they are incompatible, see eqns. 2.10a and 2.10b. That is, for a model which is exactly geometrically similar to a prototype with scale ratio,  $R$ , the gas velocity in the model should be  $R$  times that in the prototype to keep  $Re$  constant.



This will change the value of Mach number and hence be incompatible. This problem is generally solved, (in wind tunnels), by choosing a different gas in the model whose density is  $\frac{1}{R}$  that in the prototype or using the same gas and a different pressure since the speed of sound is independent of the pressure.

But in this work, Mach number values are very small. The maximum air velocity through the model bed is 1 m/sec.

$$\therefore M = \frac{1}{340} = 2.9 \times 10^{-5}$$

where sound velocity,  $A = 340$  m/sec. at room temperature.

In this case, the bed/feed inlet diameter ratio = 6.

$\therefore$  The maximum air velocity in the feed inlet = 36 m/sec and

$$M = \frac{36}{340} = 0.1 \text{ in the feed inlet}$$

which is still very small and may be expected to have a negligible effect on the flow pattern of air through packed beds.

Therefore, the principle of dynamic similarity stated by Kay and Nedderman<sup>(24)</sup> should be applicable to the systems used in this work. That is, (the flow pattern in the model should be the same as in the prototype provided that Reynolds number of each is the same). The validity of this assumption on packed beds is tested by the experiments described below.

Johnston and Thring<sup>(22)</sup> stated, that under a viscosity controlled dynamic regime, i.e.  $Re$  constant, a geometrically similar packed bed suffers no frictional wall effects. The difference in both interior and boundary surface/volume ratio is compensated by the increased velocity in the model. At equal Reynolds numbers, the gas flow patterns in the model and prototype are similar.

#### 2.4.3 Gas Maldistribution Number

Porter<sup>(25)</sup> (1982) proposed the idea of a maldistribution number,  $\phi$ , which has a finite but minimum value for the uniform flow

of gas (or liquid) through a packed bed, and that was because:

1. At the small scale (of the order of the dimensions of a packing piece) even the well distributed flow is maldistributed.

2. It is not practically possible to make an exactly geometrically similar small scale model of a large randomly packed bed such that each packing piece is in the same position and orientation in both systems. However, the maldistribution factor should take care of the bed randomness.

The idea of this maldistribution number  $\phi$ , which is defined in equation 2.2, forms the basis of this work where the ability of  $\phi$  to take care of the randomness of the bed is tested.

## 2.5 Conclusions

Mass transfer through packed beds shows that it is of great importance to provide very uniform gas and liquid distribution in large diameter packed columns. Although considerable work has been done on liquid distribution, nothing has been on gas distribution except the work of Zaytoun<sup>(17)</sup>, which provided the starting point for this work.

There is need for a greater understanding of gas maldistribution in packed columns, and in particular for a method which will permit the design and evaluation of distributors for large diameter packed columns. One approach to this is described below.

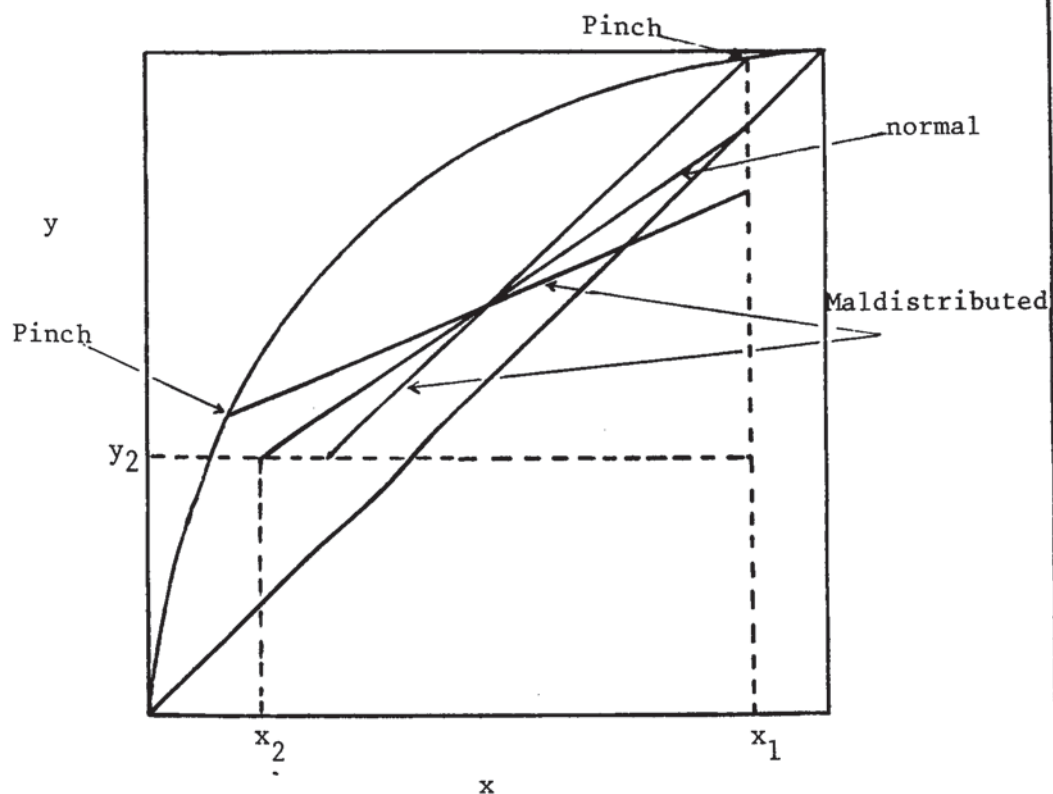


Fig. 2.1 McCabe Thiele diagram for a part of packed bed  
operating under conditions of maldistribution  
Not Equal Liquid Rates

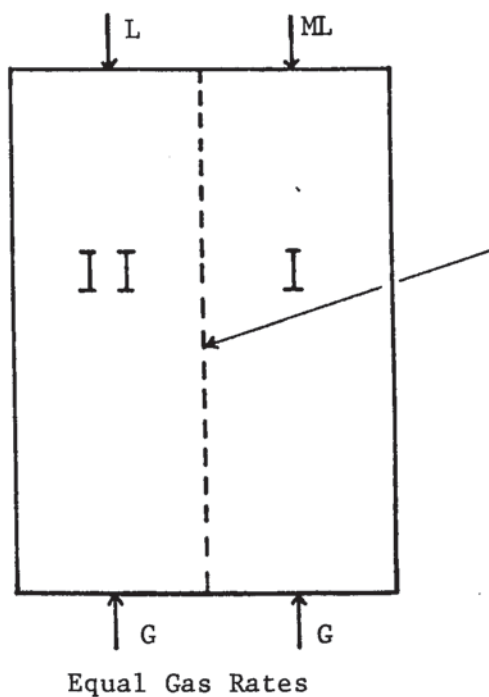


Fig. 2.2 Simple Channelling Model (after Mullin)

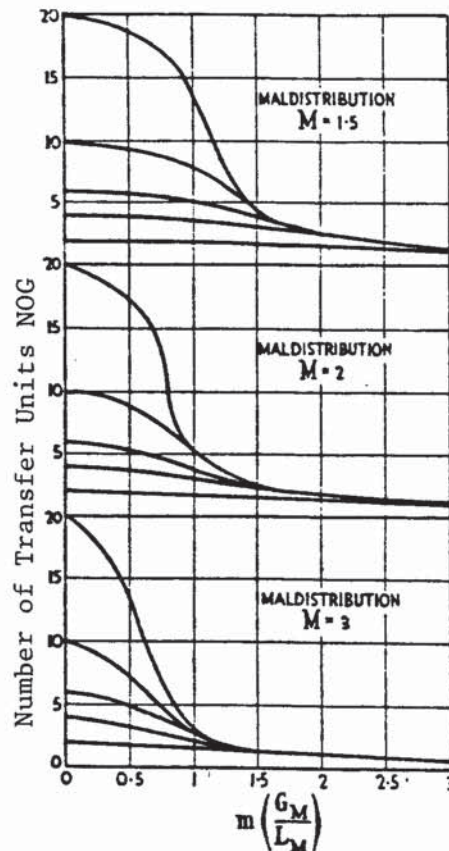


Fig. 2.3 The effect of various degrees of maldistribution on the number of transfer units. (After Mullin<sup>(1)</sup>)

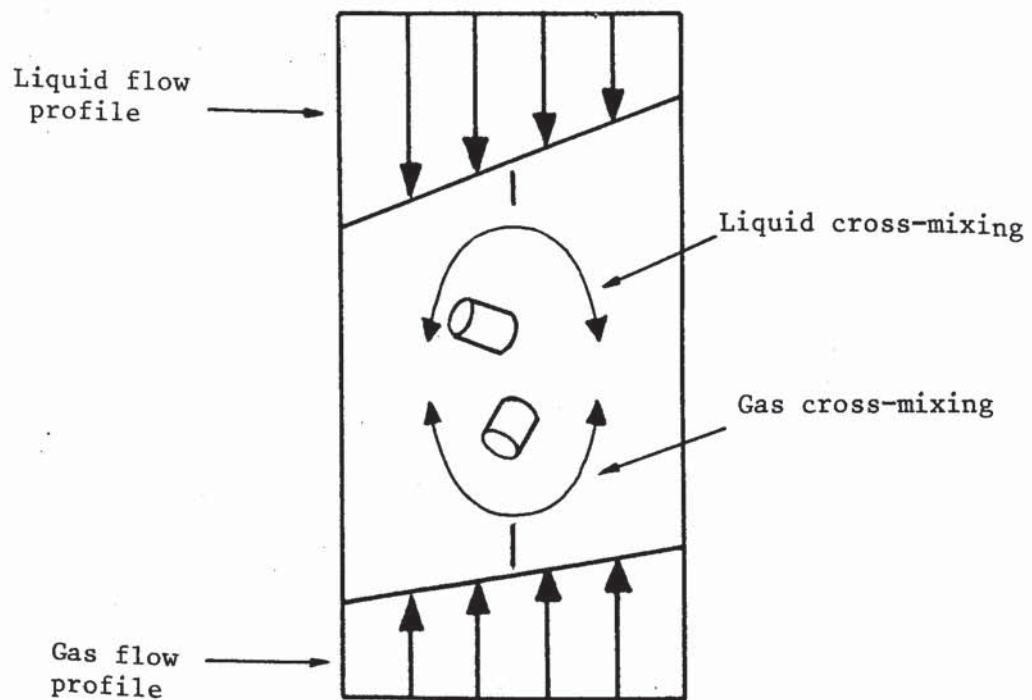


Fig. 2.4 The effect of cross-mixing on maldistribution (Huber<sup>(2)</sup>)

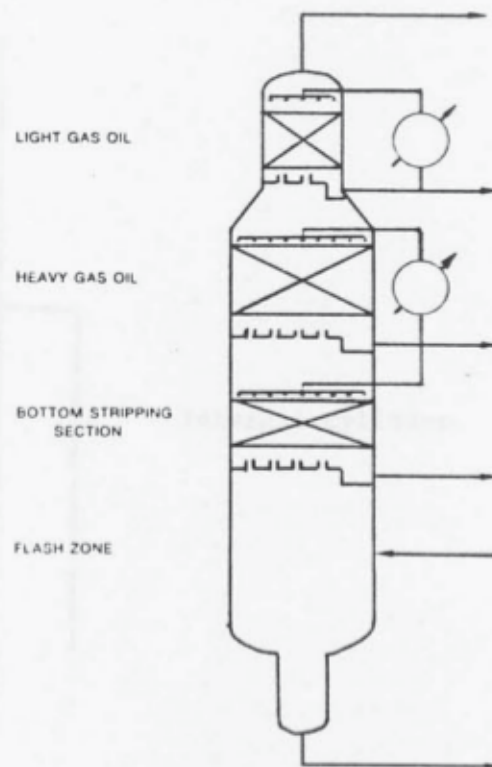


Fig. 2.5    A typical vacuum crude fractionation tower



Illustration removed for copyright restrictions

Fig. 2.6    Ladder type liquid distributor



Illustration removed for copyright restrictions

Fig. 2.7    Weir type liquid distributor

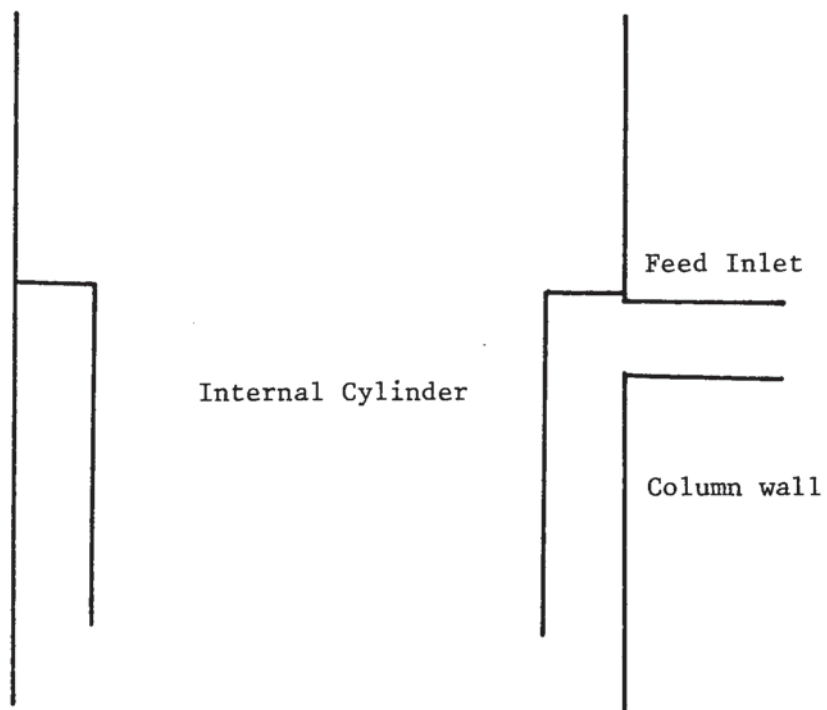


Fig. 2.8 Internal Cylinder gas distributor used with tangential feed inlets

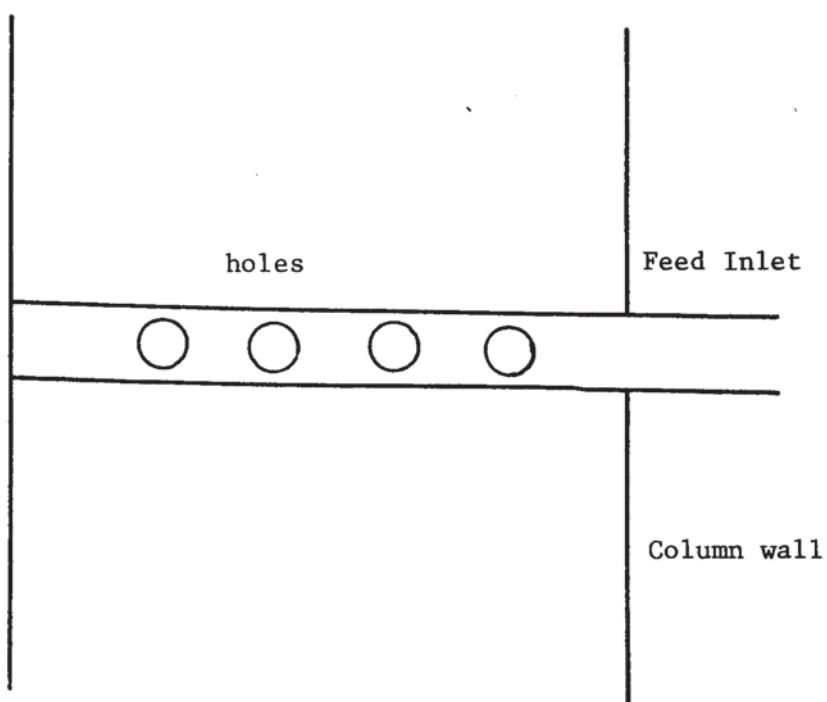


Fig. 2.9 Extended Feed Inlet gas distributor used with radial feed inlets

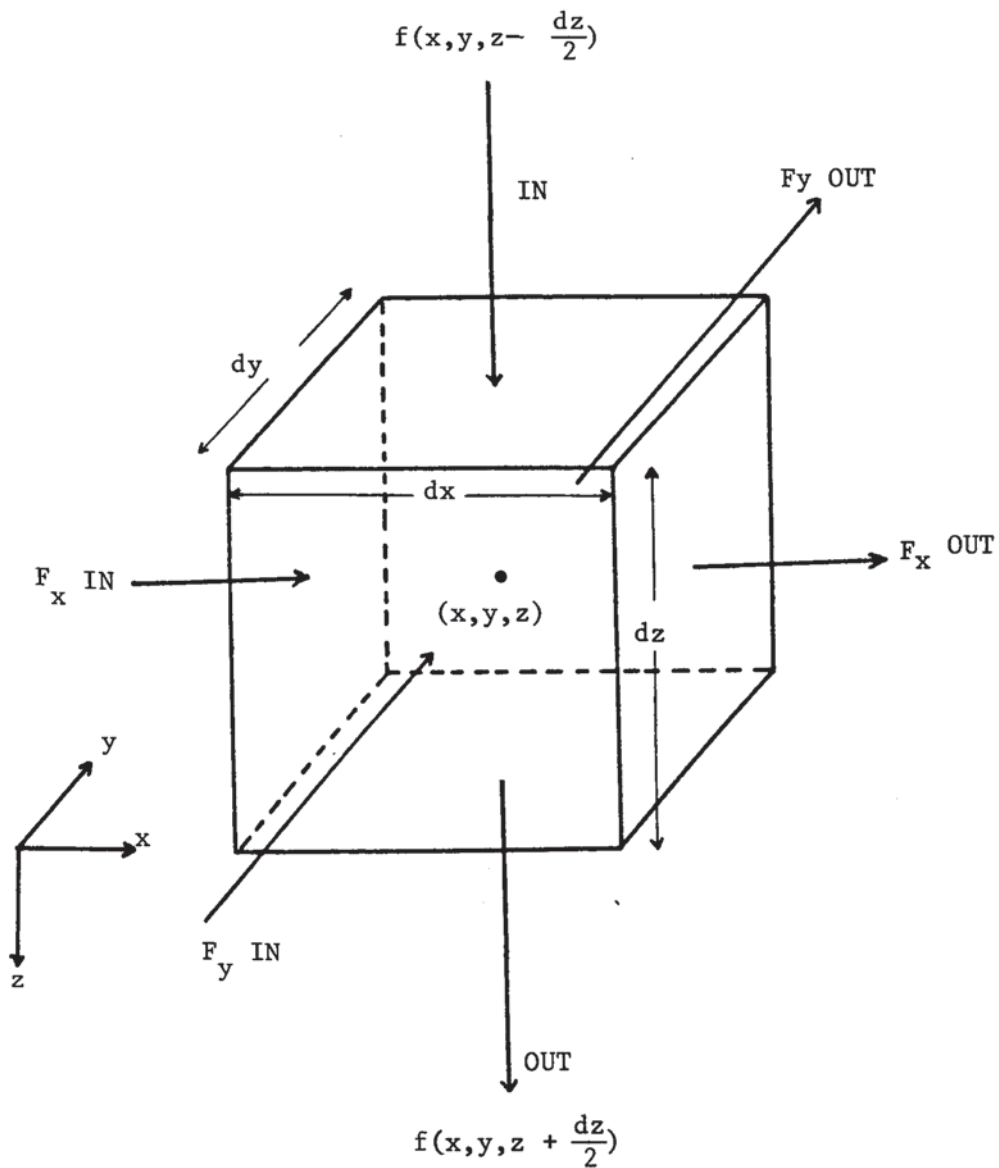


Fig. 2.10    Liquid flows into and from an elemental volume  
(Porter and Jones<sup>(8)</sup>)



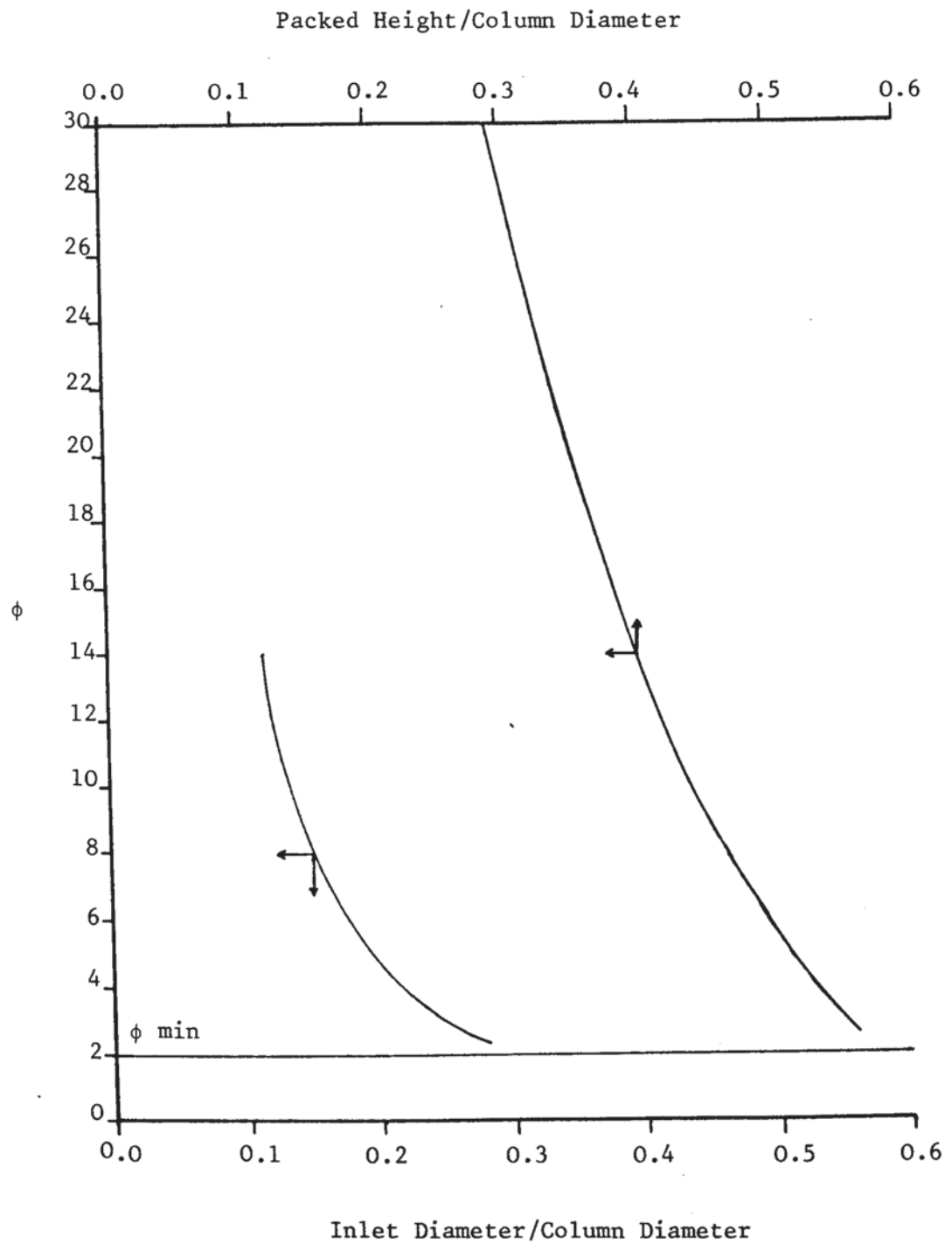


Fig. 2.11 Variation of  $\phi$  with packed height and inlet diameter (Zaytoun<sup>17</sup>).

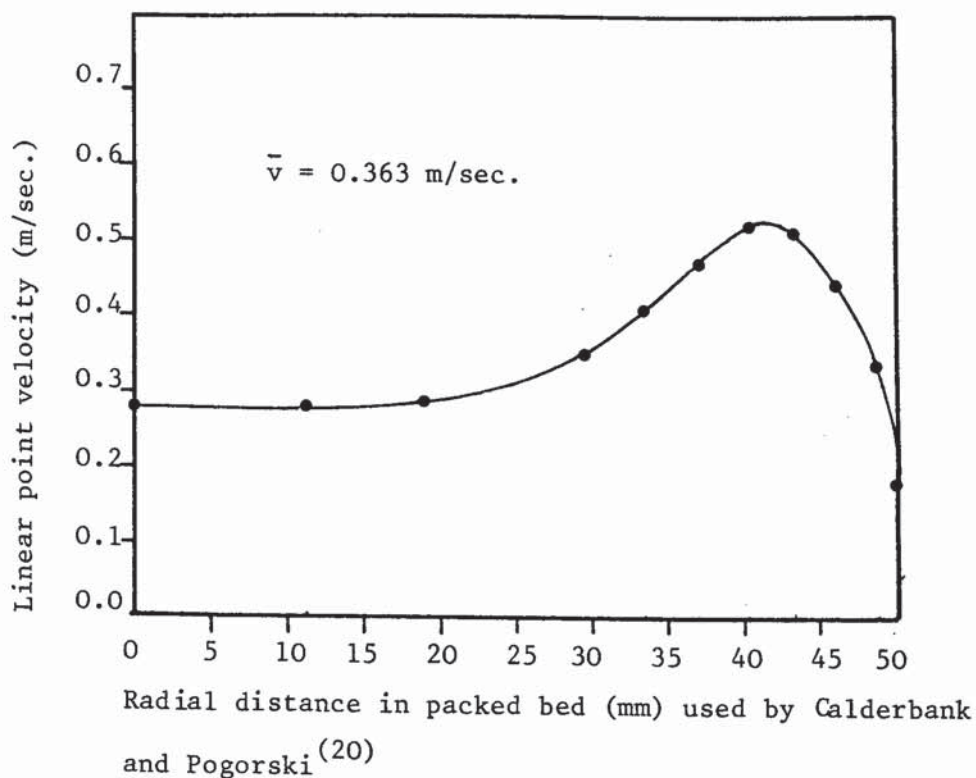
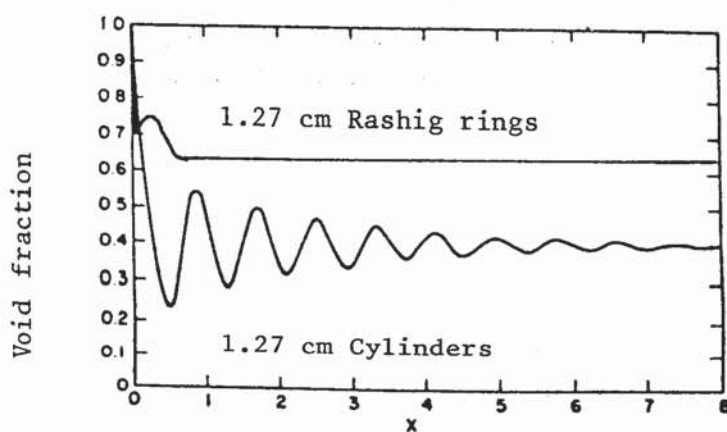


Fig. 2.12    Radial velocity profile in packed bed for  
12.7 mm diameter spheres



Distance from the wall in particle diameters

Fig. 2.13    Void fraction versus distance  
from the wall.

(Results obtained by Roblee et al.<sup>(21)</sup>  
in a 17 cm diameter column.)

SECTION III  
APPROACH TO THE PROBLEM

It was noted above that the packed bed itself acts as a gas distributor which increases the uniformity of gas distribution.

In this work, the proposal which is investigated is that a gas distribution device may be evaluated in terms of the depth of packing required to reach the best possible uniform gas flow, i.e. a good distributor requires short packed depth; a poor distributor requires large packed depth.

Following Zaytoun<sup>(17)</sup>, the problem of point to point variation in gas flow, is dealt with by defining a Maldistribution Factor which is to have a minimum value for the most uniform flow and a much larger value for a badly distributed flow. The Maldistribution Factor is based on gas velocities leaving the top of the packed bed because these are easily measured. Thus gas distribution devices may be compared in terms of a variation in the maldistribution factor with packed depth.

As the work is directed towards solving the real need for distribution evaluation in large diameter columns, it is necessary to investigate the validity of using dynamic similarity to design experiments on laboratory models of full-scale columns.

This is followed by an investigation into the fluid dynamics of maldistributed gas flow and the work concludes with the development of new low pressure drop distributors for large diameter packed beds.

The main objective of this work can be listed as follows:

- (1) Experimental investigation of point to point flow variation in gas emerging from top of a deep packed bed with a uniform inlet distribution.
- (2) Definition of the Maldistribution Factor  $\phi$ , and determination of  $\phi_{\min}$ , the minimum value of maldistribution factor for a well-distributed gas flow.
- (3) Experimental investigation of the variation of  $\phi$  with packed depth as a method of evaluating the quality of feed gas distributors, and simultaneously to investigate whether at Reynolds number dynamic similarity in geometrically similar columns of different sizes, the value of  $\phi$  is the same.
- (4) Statistical interpretation of the results by multivariable analysis of the variance to investigate the validity of scale-up rules and provide more insight into the meaning of the maldistribution factor  $\phi$ .
- (5) An experimental investigation of the fluid mechanics of maldistributed gas flow.
- (6) Simple theoretical models of maldistributed gas flow so as to:
  - (a) Support scale-up rules.
  - (b) Identify packed column designs at risk; i.e. subject to gas maldistribution.
- (7) Evaluation of existing gas distribution devices in an "at risk" packed tower. The example chosen is the pump around section of 7.3m diameter vacuum crude distillation column.
- (8) Development of new low pressure drop gas distributors for large towers.

## SECTION IV

### EQUIPMENT

#### Introduction

In all of the experiments, air was caused to flow through packed beds of various designs. This section describes the equipment used to provide the air supply and measure the rate of flow. It also describes the techniques used to measure point velocities and to reduce noise level.

#### 4.1 Air Supply Equipment

Air was pumped under atmospheric pressure into the packed bed from a 10 horse power fan through a 152mm inside diameter QVF pipe. The flow could be controlled by means of two butterfly valves of 152 mm inside diameter, and measured by a "Dall Tube" flowmeter of the same pipe diameter, see Plate 4.1.

In order not to exert pressure on the fan, the excess air was released through a specially designed silencer to atmosphere. The piping system and the silencer were supported by a bench built of 38 x 38 mm steel handy angle bars. The QVF tubes are fitted to the bench by specially designed wooden attachments and secured by belts to the attachments, see Plate 4.1, and the whole system is on the same level as the fan exit. The fan exit is joined to the QVF pipe by a 600 mm long and 180 mm inside diameter flexible tube to absorb the vibration and the start-up shock of the fan. Air enters the test column through changeable nozzles of different sizes.

#### 4.2 Measurement of Flow, Velocity, and Pressure

In the experiments described in this work, special techniques were used to measure total flow rates, point velocities and pressure.

These are described below.

#### 4.2.1 Flow Measurement by a Dall Tube

The Dall Tube was chosen due to its short length, 152 mm, and low pressure loss. The Dall tube, see Fig. 4.1, consists of a short length of parallel lead-in-pipe followed by the converging upstream cone and the diverging downstream cone. A small gap between the two cones allows the fluid to fill the annular space between the liner and the case. The throat pressure is transmitted to the annular chamber and then via a pressure pipe to the measuring instrument.

The Dall Tube has the desirable features of high measured pressure drop (similar to an orifice)<sup>(26)</sup>, and low permanent pressure loss (pressure difference between fully developed upstream flow and fully developed downstream flow), similar to, and sometimes better than, a venturi. These apparently inconsistent advantages have been checked experimentally but are not fully explained theoretically. The permanent pressure loss of a Dall tube is of the order of 50% or less of that of a venturi tube with the same pressure drop.

However, Kent<sup>(27)</sup> summarizes the important advantages of the Dall tube over other kinds of variable-pressure-drop flow meters; the flow nozzle and the venturi tube, and these advantages are as follows:

(a) Lowest head loss; the pressure loss of the Dall tube varies between  $\frac{1}{2}$  and  $\frac{1}{3}$  of that of the long-pattern venturi tube.

(b) Easier and cheaper installation than a venturi tube. The length and weight of the Dall tube are about  $\frac{1}{2}$  those of a short-pattern venturi tube.



(c) Lower first cost than venturi tube owing to economies in materials and machining.

The Dall Tube flow was calibrated against the flow calculated from the velocity profile measured by a hot-wire anemometer, see Appendix 1A.

#### 4.2.2 Velocity Measurement by a Hot-Wire Anemometer

Hot-wire anemometers are commonly made in two basic forms; the constant-current type and the constant-temperature type<sup>(26)</sup>. The difference between these types is primarily in the electric circuits.

The hot-wire anemometer consists essentially of an electrically heated, fine wire (generally platinum) exposed to the gas stream whose velocity is being measured. An increase in fluid velocity increases the rate of heat flow from the wire to the gas, thereby tending to cool the wire and alter its electrical resistance. In a constant-current anemometer, gas velocity is determined by measuring the resulting wire resistance; in the constant-temperature type, gas velocity is determined from the current required to maintain the wire temperature, and thus the resistance, constant.

For the measurement of average (steady) velocities, the constant-temperature mode of operation is often used (as in this work). Another factor may effect the rate of heat flow from the wire, namely the gas temperature which effects the convection film coefficient, therefore, to get more accurate work, a given hot-wire probe must be calibrated in the fluid in which it is to be used. For this purpose the anemometer must be set to zero by putting the probe in a hole in which the fluid velocity is zero, where both the



probe and the hole have the same temperature as the fluid stream.

The hot-wire anemometer can accurately measure velocities from about 0.15 m/sec. to supersonic velocities.

#### 4.2.3 Modification of the Hot-wire Anemometer

A 3000-ETA hot-wire anemometer was used in this work, (see Fig. 4.2), to measure point velocities above the bed. Some modifications were made to the anemometer to suit the measurement procedure.

This type of hot-wire anemometer is compact, hand held and measures air velocities from 0.1 m/sec up to 15 m/sec with a semi-logarithmic scale. The probe is situated in the middle of a 10 mm diameter hole in an arm 15 mm diameter and 160 mm long which can be extended to give a maximum length of 850 mm. The arm is connected to the meter by 2m, telephone-type, probe wire. The arm should be positioned such that the hole is exactly in-line with the air stream. This design does not always suit the sampling procedure followed in this work, because, sometimes it is necessary to hang the arm from a position high above the column to read point velocities just above the packed bed inside the column with minimum or negligible obstruction to the air stream emerging from top of the bed. So, the ordinary extension to the arm was removed and the probe-cable was inserted inside a 6 mm diameter steel extension 1.5m long and fitted to the arm in a way such that the arm could be put either in-line with, or perpendicular to this new extension. The other end of the extension was clamped to a spirit level to ensure accurate positioning of the probe, and graduations were engraved on this new long, convenient arm, (see Plate 4.2). Also, the power was supplied through a stabilized transformer instead of a battery power supply

which decays after 20 hours work. The use of stabilized DC supply is very essential, because in an ordinary transformer, the output DC has a resonance which is detectable on the hot-wire anemometer and a distorted air velocity reading is obtained. The stabilized transformer solves this problem, where the output power is smooth.

#### 4.2.4 Measurement of Pressure by a Micromanometer

A micromanometer was used to measure very small pressure variations throughout the packed bed. The MDC micromanometer, shown in Plate 4.3, was chosen because it is a sensitive pressure measuring device capable of measuring gas pressures down to  $10 \text{ dynes/cm}^2$  for Full Scale Deflection, (F.S.D.). This is equal to 0.004" wg (0.1 mm). Its accuracy as a percent of F.S.D. of the instrument is 0.1%.

To measure the pressure difference, the micromanometer should be adjusted to read zero on the scale before any connection. The pressures to be measured are taken to the measuring head via two small hose entries on the front panel of the instrument, one hose entry for the low pressure side, the other for the high pressure side. The measuring head consists of two symmetrically arranged cavities separated by a metal diaphragm. The diaphragm, together with a fixed electrode on either side, forms two condensers. Movement of the diaphragm due to a pressure difference in the cavities causes a variation in the capacitance between it and the adjacent electrode, thus unbalancing the voltage across the circuits. These voltages are compared by a differential voltmeter, and the difference is amplified and is shown on a meter calibrated directly in pressure. The diaphragm in the measuring head will respond to sine wave pressure variation up to 200 pulses/sec. or more, but the long length of the PVC tubes (3m each) acted as attenuators.

To read static pressure at a point in the packed bed, the high pressure entry is connected by the 3m long PVC tube 2mm inside diameter, the other end of the tube is connected to the static pressure side of a pitot-static tube whose circumferential ring of small holes is at that point. The low pressure side of the manometer is left open to atmosphere.

To measure air velocity at a point in a swirling stream or straight stream below packed beds where the velocity varies significantly from point to point, the low pressure side is connected to the static pressure side of a pitot-static tube and the high pressure side is connected to a 0.5 mm inside diameter steel tube which measures the total pressure. The open end of the tube was put as near as possible to the circumferential ring of holes on the pitot-static tube, they should not be very close to each other to avoid interfering between them, (see Plate 4.4).

#### 4.3 Noise Reduction by the Development of a Special Silencer

A major source of noise in process plant is from moving fluids, particularly gases. In this work, noise comes from the vent to discharge excess air, from the blower and from the motor.

The noise generated by flowing of air in the blower inlet was reduced by lining it with 2 cm thick sponge. The mechanical noise was reduced by sticking sponge all-over the blower. At the vent, noise caused by the high velocity discharge is created by the shearing action of turbulent air mixing with the essentially stationary outside air.

A simple silencer was designed to achieve two aims:

- (1) To reduce the level of the noise by orienting the discharge vertically to direct the noise away from the work area, and

to keep the noise source as far away as possible.

(2) To act as an attenuator to absorb the fluctuation of the flowing air, where it has been noticed that without this silencer the air velocity, detected by the hot-wire anemometer, was fluctuating above the bed with frequency of around 0.5 cycle/sec.

Air enters the silencer through a 152 mm diameter butterfly control valve. The silencer itself, (Figure 4.3), consists of the following parts:

(a) Two QVF pieces 300 mm inside diameter flanged together to form a pipe 2m long standing vertically on a bench and closed at the bottom. This part acts as a reservoir to absorb the flow fluctuation.

(b) A perforated 12 mm thick PVC plate, with holes 5 mm in diameter and 50% free area; (the total free area equals the silencer outlet area and double the inlet area to reduce the likelihood of regenerated noise). Each hole will act as a pipe to generate some pressure for uniform upstream flow

(c) A QVF reducer flanged to the part described in (a) holding the perforated plate described in (b) between the flanges, and flanged from above to a 200 mm diameter QVF pipe, 1 m long, to the exit which was made as high as possible.

By this special design, the noise level is reduced to less than half of its level without the silencer, and the air emerging from the bed is flowing smoothly.



Aston University

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Fig. 4.1      Cross-section in a Dall Tube



Aston University

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Fig. 4.2      Ordinary Hot-wire Anemometer

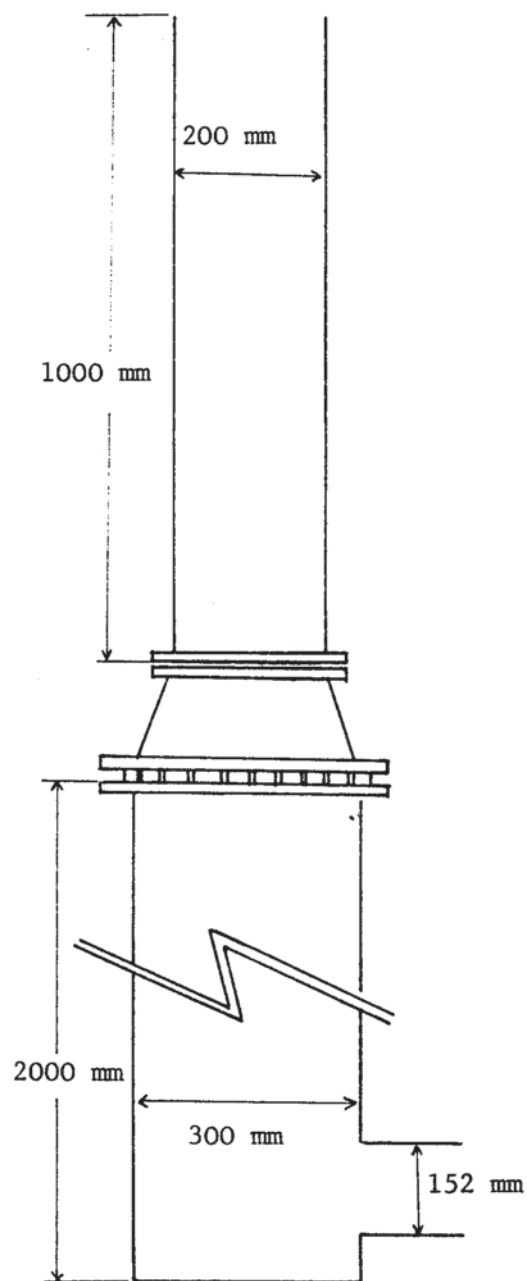


Fig. 4.3 Diagram of the Specially Designed Silencer

PLATE 4.1



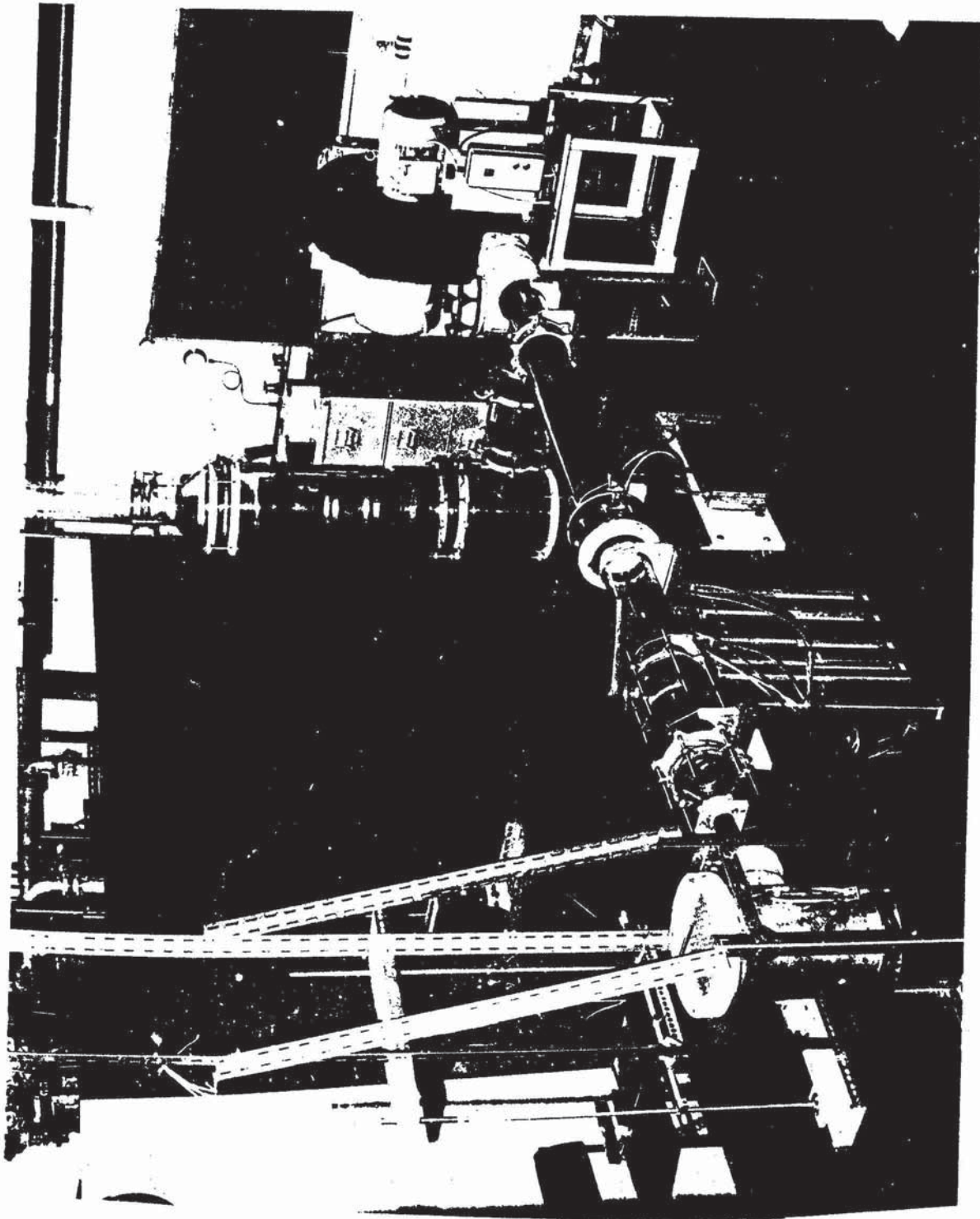


PLATE 4.2

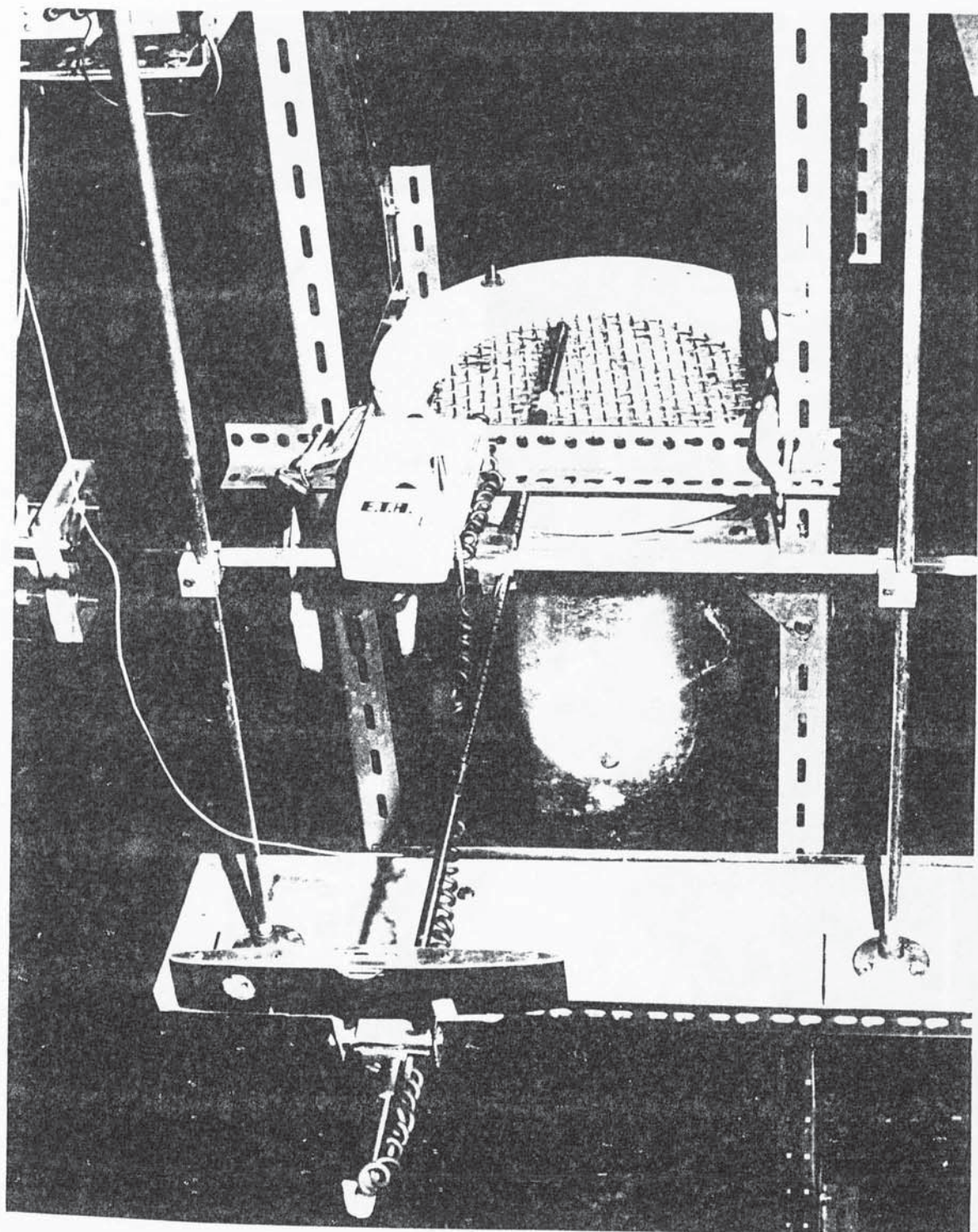


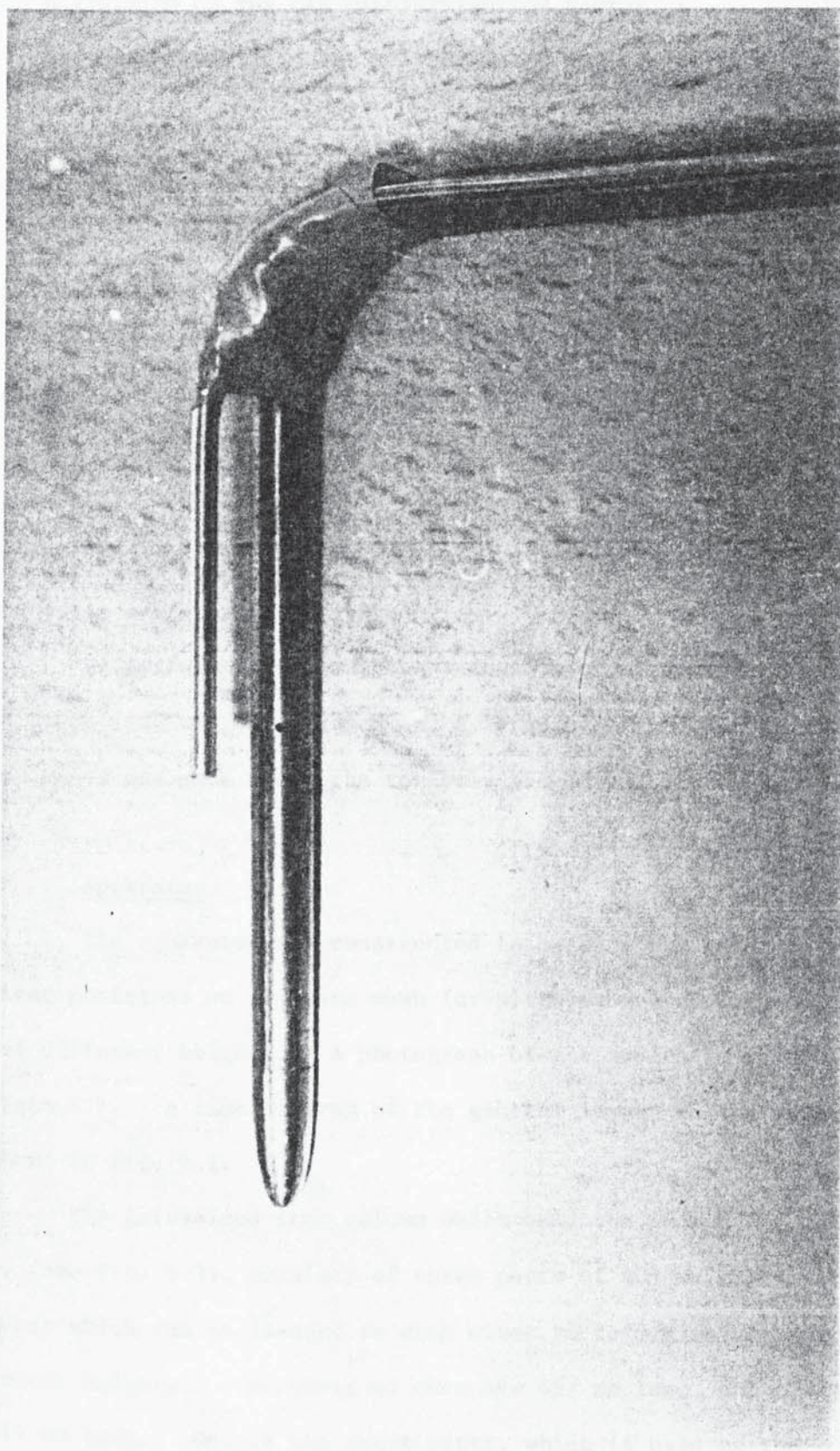
PLATE 4.3





PLATE 4.4





## SECTION V

### POINT TO POINT GAS VELOCITY VARIATION, DEFINITION OF THE GAS MALDISTRIBUTION FACTOR, $\phi$ AND DETERMINATION OF THE MINIMUM VALUE OF $\phi$ , $\phi_{\min}$ .

#### Introduction

There are two objects in the work described in this section;

(1) An experimental investigation of point to point flow variations in gas emerging from the top of a deep packed bed with a uniformly distributed gas entering bottom of the bed.

(2) Definition of a gas maldistribution factor,  $\phi$ , and determination of  $\phi_{\min}$ , the minimum value of the maldistribution factor for a uniformly distributed flow.

To perform this work, the technique of measuring gas point velocities was established and a large number of point velocity measurements was made above the randomly packed bed.

#### 5.1 Apparatus

The apparatus was constructed to measure the point velocity at fixed positions on a square mesh (or pitch) above a randomly packed bed of different heights. A photograph of the apparatus is shown in Plate 4.1. A line diagram of the general layout of the apparatus is shown in Fig. 5.1.

The galvanized iron column which held the packed bed under test, (see Fig. 5.2), consists of three parts of 305 mm inside diameter which can be flanged to each other to form a column of different heights. Two parts of them are 457 mm long, the third is 914 mm long. One of the short parts, which is used as the

bottom part, was fitted with one flange, the other two parts were each fitted with two flanges. All the flanges are 308 mm inside diameter to fit the outside of the column, and 38 mm wide and 6 mm thick. Four wooden rings lined with galvanized iron were made, with an inside diameter of 305 mm, 38 mm wide and 25.4 mm thick. These rings enabled a series of short bed heights to be obtained. A 9.5 mm square mesh made from 1 mm diameter wire was used as a packing support. A similar mesh was put over the bed to prevent the packing from floating at high air velocities. A 1.5 mm thick galvanized iron ring of 10 mm width with six attachments to the column shell was designed to hold securely the support wire mesh. The air was fed radially to the column through a 152/51 mm diameter QVF reducer, the reduced end was extended by a 457 mm long PVC pipe 51 mm inside diameter, which is flanged to the column at the feed inlet.

An elaborate gas distributor was designed, (see Fig. 5.3), to give an even gas distribution below the bed. It consists of three parts;

- (1) A galvanized iron cylinder of 254 mm inside diameter and 355 mm long, fitted with a flange 305 mm outside diameter at the top end. 8 slots, each 50 mm long and 30 mm deep, were made at equal distances at the bottom end of the cylinder. This was inserted inside the bottom part of the column which is blocked at the bottom end by a PVC plate such that the 8 legs between the slots are fitted securely to the blocking plate and the flange is positioned just above the feed inlet. A sealant was used to close any gap between the flange and the inside surface of the column. A small wooden screen was put in the annular space between the column



shell and the cylinder near the feed inlet to direct the feed in one direction, forcing the air to swirl in the annular space. The swirling air, after travelling a long distance from the feed inlet down to the slots at the base, which is about 6 times the feed pipe diameter, is distributed equally around the base of the cylinder. This equally distributed swirling air which enters the cylinder through the 8 slots of equal areas, will swirl uniformly inside the cylinder travelling upwards a distance of 203 mm to ensure a complete mixing before hitting four baffles measured 152 x 127 mm fitted to each other to form a cross baffle whose duty is to straighten the flow of the air before leaving the cylinder.

(2) Air leaving the cylinder enters the next part of the distributor which is a uniform bed of knitted mesh, 305 mm diameter and 100 mm high, held between two pieces of 300 mm diameter wire mesh, and fitted to the flange of the cylinder.

(3) Above the knitmesh, a perforated PVC plate, 380 mm diameter and 13 mm thick, is fitted. The 729 perforations are distributed on 1 cm square pitch, each 5 mm diameter. Each perforation will act as a tube; (length more than diameter), to produce a high pressure drop for even gas distribution. This plate, with 20% free area, is flanged between the bottom part and the main part of the column which contains the packed bed. In a preliminary check, the velocity of the air emerging from each of the holes was measured and found to be constant.

The support ring and the support wire mesh were fitted to the main part of the column at a distance of 150 mm above the perforated plate, so as to ensure a complete mixing of the air jets before entering the bottom of a bed randomly packed with 25.4 mm plastic pall rings to different heights.

This design of apparatus ensured a uniformly distributed air flow entering the bottom of the bed. This is subsequently referred to as the "Ideal distributed case".

On the other hand in the "Maldistributed case"; (where the feed enters the bed as it is, without passing through any kind of distributor), the distributor is removed and the feed entered radially to the column through the feed pipe of a relatively small diameter; (51 mm).

A modified hot-wire anemometer, described in Section 4.3, was used to measure the point velocities above the bed.

## 5.2 Gas Velocity Distribution Measurement Technique

The point velocities above the bed were measured by a modified hot-wire anemometer, with the arm in-line with the extension, (see Plate 4.2). A scaled extension was clamped at right angles to a fixed scaled rail, such that the anemometer's arm can be moved to any required position across the bed by moving the scaled extension in the x and y directions along the fixed rail.

Two more important factors should be known to establish the complete proposed technique of measuring the gas velocity distribution above a packed bed;

(1) The first is how far the anemometer probe should be situated above the bed to measure point velocities of different magnitudes.

It was found experimentally for a large range of gas flow rates, that a measured velocity of air at a specific point in the surface of the bed depends on the distance above that point that the anemometer's probe is situated. As the probe departs from the bed, the measured velocity decreases until reaching a position at which

the velocity remains constant over a short range of distances, after this range of distances, the measured velocity either increases or decreases. The probe was situated within this range of constant velocity. Also, it was found experimentally that this range of distances does not depend on the gas velocity at that point, but it does depend on the size of the packings.

The results show that for a wide range of air flows and for plastic pall rings of size 16 mm, 25.4 mm and 50.8 mm, the probe should be at a distance of 35 mm, 55 mm and 110 mm above beds packed with these different sizes respectively.X

(2) The second important thing was to minimize the number of point velocity measurements to give a reliable estimation of the maldistribution of the gas emerging from the top of a packed bed. This depends on the configuration of the points at which the air velocity is measured and on the definition of a maldistribution factor which gives an estimation of gas maldistribution in the packed beds.

It is a fact that in the packed beds there exist a variation in the magnitude of gas velocities from point to point across the bed due to various pathways that the gas follows through the packing. These pathways change when the bed is repacked. This fact is observed very clearly during the course of this work. This means that a full scanning of the velocity variation is required to obtain the most reliable picture of the distribution of the gas.

The way to do this scan is to measure the gas velocity at close enough, equally spaced positions.

Thus, the point velocity measurements were taken on a square mesh at a suitable distance above the packed bed.

X See additional comment on P.54.



### 5.3 Definition of Maldistribution Factor $\phi$ and Determination of $\phi_{\min}$

In this part of the work, two aims were achieved, the definition of a Maldistribution Factor and the minimum number of point velocity measurements required to obtain a reliable estimation of this maldistribution factor.

An appropriate maldistribution factor must have two essential properties:

(1) It varies significantly according to the quality of gas distribution.

(2) It has a minimum value for a well distributed gas flow and a much larger value for a maldistributed gas flow.

An attempt has been made to define a factor which gives an idea about the uniformity of the distribution based on the variance,  $\sigma^2$ , where

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (\bar{V} - V_i)^2 \quad \dots (5.1)$$

$n$  : The number of positions at which point velocity is measured

$\bar{V}$  : Value of the average point velocity, (m/sec.)

$V_i$  : Value of point velocity at  $i$ th position (m/sec)

Equation 5.1 can be rewritten as:

$$\sigma^2 = \frac{\bar{V}^2}{n-1} \sum_{i=1}^n \left(1 - \frac{V_i}{\bar{V}}\right)^2 \quad \dots (5.2)$$

The summation term is taken as the "Maldistribution Factor",  $\phi$ ,

i.e.

$$\phi = \sum_{i=1}^n \left(1 - \frac{V_i}{\bar{V}}\right)^2 \quad \dots (5.3)$$

Equation 5.2 states the mathematical formulation of  $\phi$  which is the sum of deviations of point velocities from the average value of the point velocity. The properties of the maldistribution factor  $\phi$  are estimated experimentally in this section of the work.

#### 5.4 Experimental Work and Results

The experimental work was carried out on two extremely different beds packed with 25.4 mm plastic pall rings; a well-distributed bed and a maldistributed bed. The well-distributed bed, which represents an "Ideally distributed case", that is, the feed is uniformly distributed, is packed to a depth of 1220 mm, which is much greater than the bed diameter, (305 mm). The badly distributed bed was a shallow one with a packed depth, (51 mm), much less than the diameter, (305 mm), when air is fed through a relatively narrow pipe, (51 mm in diameter), without any distribution.

The first aim of these experiments is to evaluate  $\phi$  for a well-distributed bed and at extremely maldistributed conditions so as to define the minimum value that  $\phi$  can have and to find out the reliability of  $\phi$  for describing gas maldistribution; the more the difference between  $\phi_{\min}$  and  $\phi$  for a maldistributed bed, the more the reliability of this maldistribution factor.

The second aim is to minimize the number of point velocity measurements required.

Different numbers of point velocity measurements were taken at a distance of 5 mm above these two beds, the beds were then repacked several times, and each time the bed was repacked, the

same numbers of point velocity measurements were made. This procedure was carried out at different Reynolds numbers for the ideally distributed case so as to estimate (if any) the effect of Reynolds number value on  $\phi_{\min}$ .

Examples of complete sets of observations are given in Tables 5.1 and 5.2. A computer program to evaluate  $\phi$  is given in Appendix A2. The processed data is shown in Tables 5.3 and 5.4. In these tables a different value of  $\phi$  was obtained each time the bed was repacked. These values of  $\phi$  are averaged to obtain  $\bar{\phi}$  which represents the average maldistribution of the gas emerging from the top of a packed bed.  $\bar{\phi}$  is then divided by the corresponding number of point velocity measurements so as to evaluate the minimum number of measurements required. The smallest number of measurements which produces a  $\bar{\phi}/n$  ratio nearest to that ratio obtained from the largest number of measurements, is considered as the minimum number required to obtain a reliable evaluation of  $\phi$ .

## 5.5 Discussion of the Results

Table 5.3 summarizes the results obtained for a deep packed bed which is fed by uniformly distributed air. The results show that for 161 point measurements,  $\bar{\phi}/n = 0.1025$  and for 100 point measurements,  $\bar{\phi}/n = 0.097$ , which are 2.3% and 7.6% below that obtained for 649 point measurements respectively. Therefore 161 point measurements could be considered as the minimum number that gives a reliable estimation of  $\phi$ . This conclusion could also be reached for maldistributed case, where  $(\bar{\phi}/n)$  for 161 point measurements is 2.6% less than that obtained for 649 point measurements.

The values of  $\phi$  in both of these beds change as the beds were repacked, these changes express the fact that the pathways followed by the gas through packed beds change when a repacking takes place.

Also, it could be noticed that for each bed,  $\phi$  values are scattered around an average mean within a range of  $\pm 30\%$ .

But the most important thing to be noticed is that  $\bar{\phi}$  for the maldistributed case is more than 11 times greater than that obtained for the ideally distributed case.

The average value of  $\phi$  obtained for the ideally distributed case is subsequently referred to as  $\phi_{\min}$ , the minimum value of  $\phi$ , which has a constant value at different Reynolds number values where Reynolds number,  $Re$ , is defined as:

$$Re = \frac{\rho \bar{V} D}{\mu} \quad \dots (5.3)$$

where

$\rho$  = Air density ( $\text{kg/m}^3$ ).

$\bar{V}$  = Average value of point velocity, (m/sec).

$D$  = Bed diameter, (m).

$\mu$  = Air viscosity, (kg/m.sec.).

The non-zero value of  $\phi_{\min}$  is explained by the fact that point to point velocity variations do exist in packed beds, and an even gas distribution could not be reached due to the pathways followed by the gas through the bed which cause this unavoidable maldistribution estimated by  $\phi_{\min}$ .

Comparing the observed data for a well-distributed bed and a maldistributed bed given in Tables 5.1 and 5.2, it could be noticed clearly that for a well-distributed bed, point velocities either above or below the average value, are randomly distributed all over the bed.

But for the maldistributed bed, there exists a number of point velocities concentrated in one region which all have relatively very low velocities. At the same time, there exists a concentration of high point velocities in other regions. The region near to the feed inlet contains the lowest point velocities and that opposite to the feed inlet has the highest point velocities. From this it could be concluded that  $\phi_{\min}$  represents the summation of point to point velocity variations, and  $\phi$  represents that variation plus the variations due to the regional variation of gas velocity.

It is important to note that the obstruction to the flow of the gas caused by the anemometer's probe is not effective since the relative velocity is measured. Also, because of low values of point velocities involved in the experimental investigation, and due to the dimensionless nature of  $\phi$ , the values of point velocities were measured in ft/min. instead of m/sec. to avoid large reading errors.

## 5.6 Conclusions

The following conclusions could be abstracted:

(1) Point to point velocity variations do exist in a gas emerging from deep packed beds with a uniformly distributed gas entering the bottom of the bed. These variations are due to the existence of discrete pathways followed by the gas throughout the bed. These pathways change on repacking the bed.

(2) The defined maldistribution factor,  $\phi$ , has a minimum value when the bed is deep and a well distributed gas enters the bottom of the bed.

(3)  $\phi$  has a larger value for maldistributed beds.



(4) For a packed bed,  $\phi$  has values scattered around an average value,  $\bar{\phi}$ , within a range of  $\pm 30\%$ .

(5) The minimum number of point velocity measurements required to give a reliable  $\phi$  value is around 161 for the types of beds used in this work.

## 5.7

### Summary

A technique was established to evaluate gas maldistribution above a packed bed and a maldistribution factor,  $\phi$ , was defined.  $\phi$  has a minimum value for an ideal bed; deep and fed by a uniformly distributed gas. Also,  $\phi$  has a much larger value for a shallow bed with feed introduced through a relatively small pipe to the column and in which the gas was not evenly distributed. In the next section, an investigation of the effect of packed height on gas distribution in the bed is going to be made.

### Additional Comments

1. It was found that when the anemometer is positioned above the bed, at the distances specified on p. 48, the total air flow calculated from  $\bar{v}$  differs by not more than 5% from the true air flow measured in an empty pipe. See Appendix A1.
2. There is no systematic increase or decrease of  $\phi$  with repacking. The results are presented in order of increasing  $\phi$  values in order to demonstrate the maximum variation in magnitude.



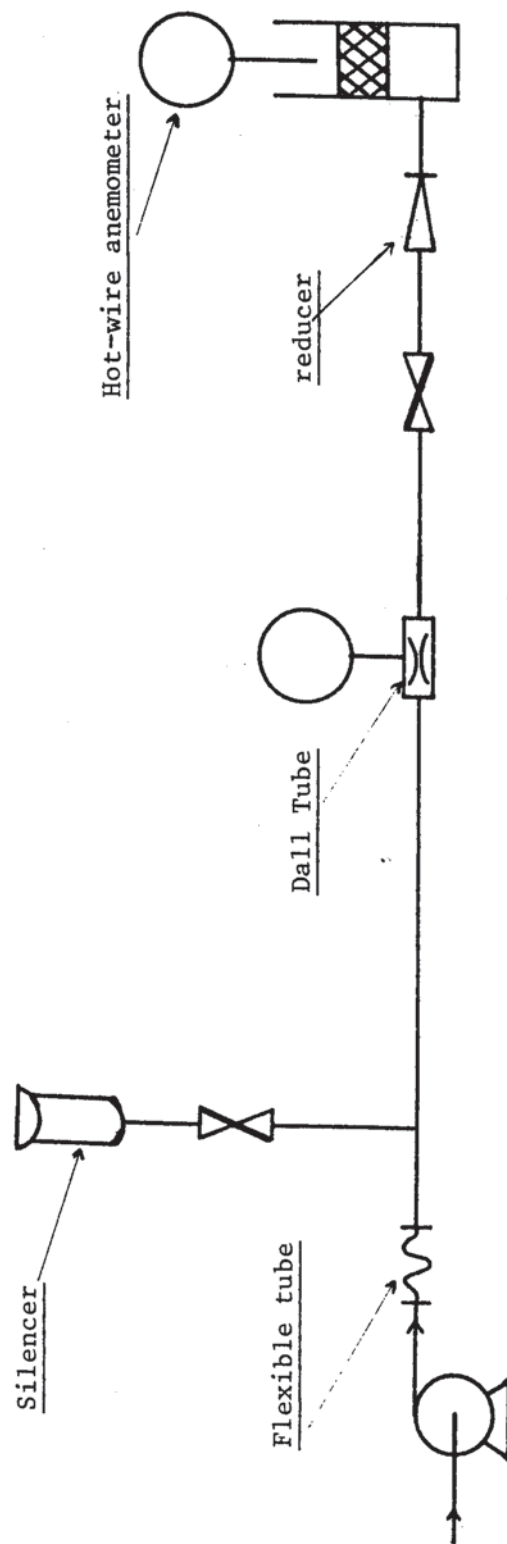
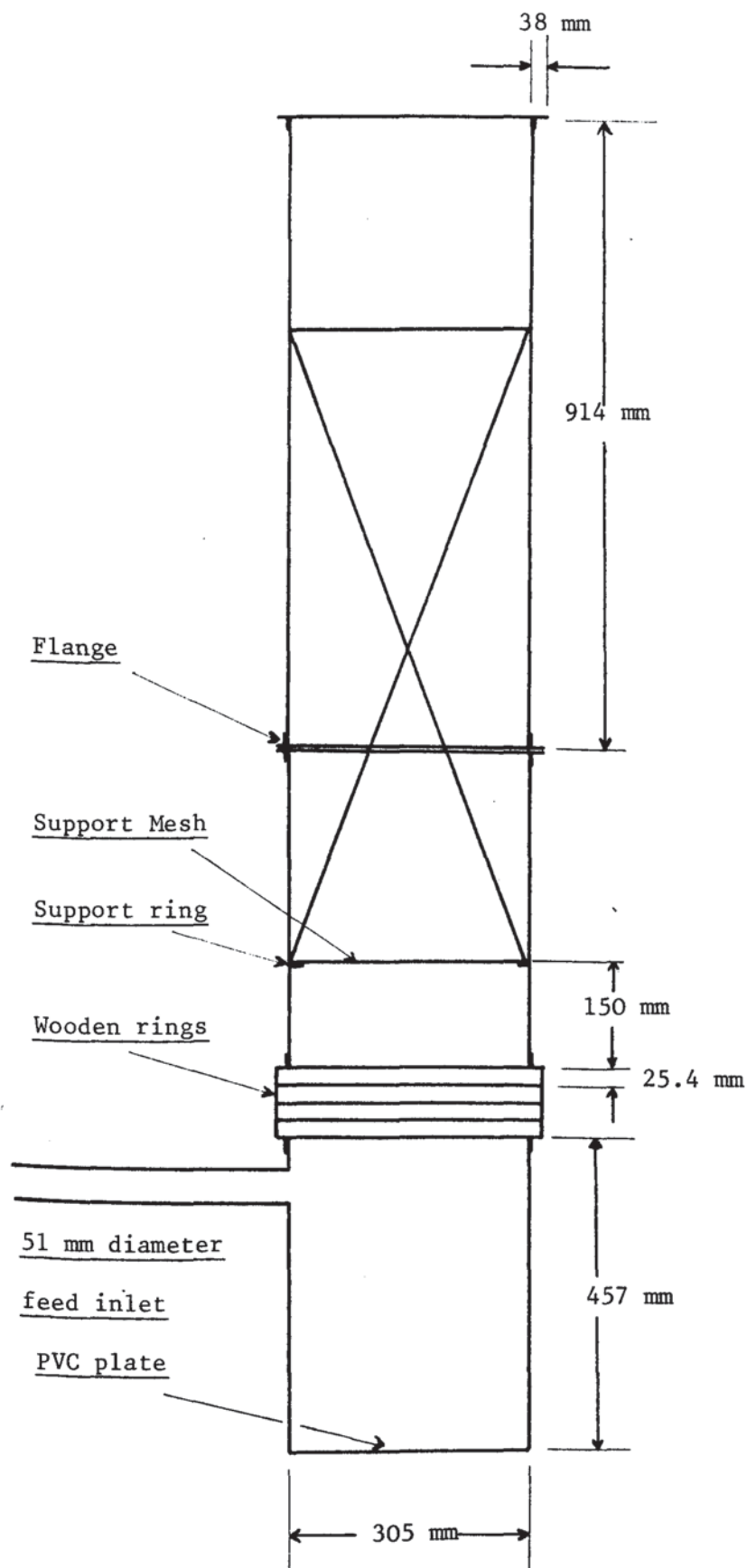


Fig. 5.1 General layout of the apparatus

Fig. 5.2    The 305 mm diameter packed column



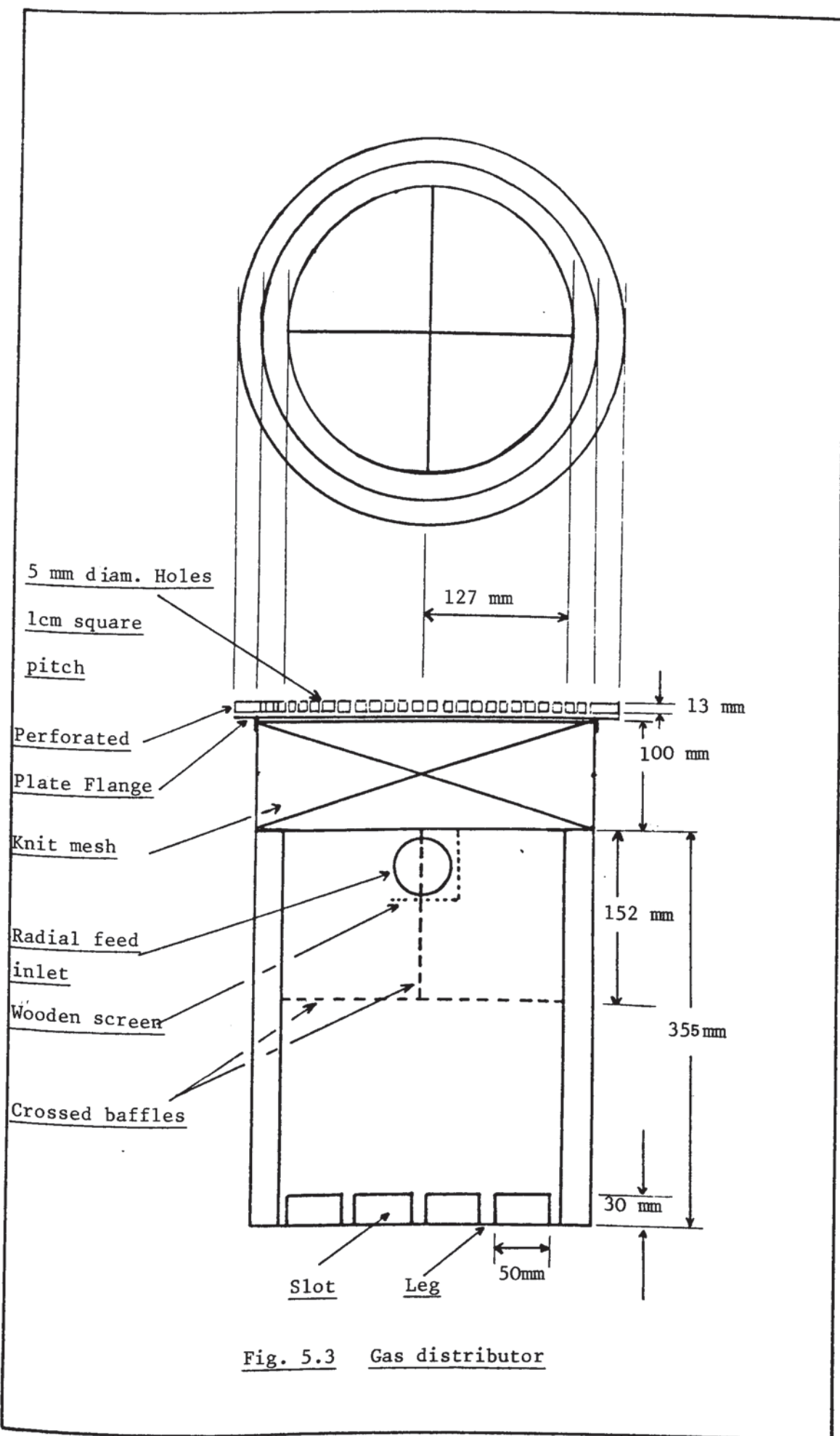


Fig. 5.3 Gas distributor

[illegible]

[illegible]

Table 5.3 Maldistribution Factor,  $\phi$ , for a 1200 mm Deep

Well-distributed Packed Bed

NOTE: SEE COMMENT 2 ON P. 54 FOR  $\phi$  VALUES

Re	n	repack	$\phi$	$\bar{\phi}$	$\bar{\phi}/n$
29300	649	1	44.9	68.14	0.105
		2	58.1		
		3	60.7		
		4	70.5		
		5	72.1		
		6	74.0		
		7	76.2		
		8	88.6		
	161	1	12.4	16.5	0.1025
		2	12.6		
		3	13.2		
		4	16.8		
		5	18.3		
		6	18.7		
		7	19.82		
		8	21.2		
	100	1	13.29	9.7	0.097
		2	13.01		
		3	12.3		
		4	9.0		
		5	8.4		
		6	7.8		
		7	7.7		
		8	6.1		
39100	649	1	55.3	6.9	0.1063
		2	58.0		
		3	68.3		
		4	75.7		
		5	76.5		
		6	80.0		



Table 5.4 Maldistribution Factor  $\phi$  for a 51 mm Deep

Maldistributed Packed Bed at  $Re = 29300$

NOTE: SEE COMMENT 2 ON P.54 FOR  $\phi$  VALUES.

n	Repack	$\phi$	$\bar{\phi}$	$\bar{\phi}/n$
649	1	650.5	768.5	1.184
	2	653.5		
	3	690.1		
	4	700.3		
	5	786.0		
	6	847.6		
	7	856.2		
	8	963.8		
161	1	132.1	185.7	1.153
	2	152.6		
	3	178.9		
	4	188.7		
	5	195.3		
	6	197.5		
	7	203.2		
	8	237.7		

SECTION VI  
METHOD OF EVALUATING GAS DISTRIBUTORS  
AND SCALE-UP RULES

Introduction

The main purposes of the work described in this section are:

(1) To develop a method of testing the quality of feed gas distributors for large diameter packed columns, based on variation of the maldistribution factor  $\phi$  with the packed depth.

(2) To determine experimentally the rules of scale-up for using laboratory models of a full-scale column to test different kinds of gas distributors.

These two aims are achieved simultaneously throughout the course of the work in this section.

Before progressing in describing this work and its result, it is necessary to make a note about the rules of scale-up.

6.1 Rules of scale-up of Packed Beds

The packed beds scale-up rules which are going to be determined experimentally in this work can be summarized as follows:

(1) At low Mach number, dynamical similarity is achieved by keeping Reynolds number constant in exactly geometrically similar packed beds.

(2) The lack of geometric similarity resulting from the randomness of the packed bed is taken care of by the use of the maldistribution factor  $\phi$ .

(3) For geometrically and dynamically similar beds, whether the gas beneath the bed is well or badly distributed, the

value of the maldistribution factor  $\phi$  is the same in both the model and the full-scale beds.

(4) The effectiveness of a gas distributor may be measured in terms of the height of packing required for the minimum value of the maldistribution factor to be achieved, i.e. good distributors require less packed height than bad distributors to attain  $\phi_{\min}$ .

That dynamic similarity is achieved by keeping Reynolds number,  $Re$ , constant needs to be confirmed experimentally because:

(a) The gas used in this work is air, which is compressible; that means the Mach effect does exist.

(b) The randomness of the packed bed, which means that a different arrangement of the packings results each time the bed is repacked.

In this work, Mach number values are very small. The maximum gas velocity through the model bed is about 1m/sec. That is Mach number  $M = 3 \times 10^{-5}$  and the maximum Mach number value in the inlet feed is about 0.1 which is still small and may have a negligible effect on the flow patterns. But the Mach number will be different in the model and the full-scale bed.

Therefore, if Reynolds number is sufficient to produce dynamic similarity, then the flow pattern in the model should be the same as in the prototype provided that the Reynolds number of each is the same. The validity of this assumption is tested by the experiments described in this section of the work.

Reynolds number is defined in this work based on the diameter of the packed bed,  $D$ , as follows:

$$Re = \frac{\rho \bar{V} D}{\mu} \quad \dots\dots (6.1)$$

The first set of experiments was performed to check that at the same  $Re$ ,  $\phi_{min}$  is the same in the model and the prototype.

However, the maldistribution factor  $\phi$  should take care of the bed randomness, but this is also tested in these experiments.

## 6.2 Apparatus

Two exactly geometrically similar columns of different size were constructed and used to check the rules of scale-up stated above. Both columns were made of 1.5 mm thick galvanised iron. The small column is considered as a model of the large one. Both the model and prototype were constructed to permit measurements of the point velocities at fixed positions on a square mesh above geometrically similar beds randomly packed with geometrically similar packings.

The model, see Fig. 6.1, consists of two equal parts, 381 mm inside diameter and 571 mm long. Air enters the bottom part radially through a 63.5 mm feed pipe whose centre-line is 381 mm above a flat closed end of this part. The top end of this part is fitted with a 482 mm outside diameter flange. An elaborate distributor, similar in principle to that described in Section 5.1, was designed. It consists of the following parts:

(1) A cylindrical part, 317 mm in diameter and 412 mm long, held to a distance of 30 mm above the flat closed bottom end by means of four legs, each 30 mm long, equally spaced around the bottom end of the cylinder and fitted to the flat bottom of the column. The annular gap is closed at the top of the cylinder at a position exactly above the feed inlet. A wooden screen is inserted in the annular space at a position near the feed inlet to cause the air to swirl in the annular space. a 190 mm deep cross

baffle is fitted inside the cylinder with its top end is positioned parallel to the top end of the cylinder.

(2) A distribution bed packed with 16 mm plastic pall rings to a depth of 95 mm. The duty of this bed is to distribute the air emerging from the top of the cylinder. A distance of 64 mm is left above this bed to enable a complete mixing of air jets emerging from the top of the distribution bed before reaching a perforated plate.

(3) A perforated PVC plate, 13 mm thick, is flanged between the bottom part and the main part of the column. A total of 1140 holes each 5 mm in diameter were made to give 20% free area. The velocity of the air emerging from each of these holes was checked and found to be constant.

A 12.5 mm wide support ring is fitted to the main part at a distance of 95 mm above the perforated plate. Two 6.3 mm diameter steel support beams were put below the support ring to hold the 9.5 mm square support mesh, made of 1 mm diameter wire, and to prevent it from being bent due to the weight of the packings. The bed under test is packed to different depths with 16 mm plastic pall rings.

This design is referred to as the "Ideal distributed case".

On the other hand, the "Maldistributed case" is achieved when the distributor is removed and the bottom of the bed under test is situated at a position just above the feed inlet, see Fig. 6.2.

The prototype whose inside diameter is 610 mm is designed with every single dimension is exactly geometrically similar to, and 1.6 times, the corresponding dimension in the model. This is true for both the ideal distributed case and the maldistributed case, see Figures 6.3 and 6.4 for these two cases respectively.

The modified Hot-wire anemometer, described in Section 4.3 was used to measure point velocity at any fixed position above the bed. The probe is situated at a distance of 35 mm and 55 mm above the model bed and the prototype bed respectively. The technique of moving the probe across the bed is similar to that described in Section 5.2.

### 6.3 The Minimum Number and Configuration of Point Velocity Measurements

A preliminary experimental investigation was carried out to estimate the minimum number of point velocity measurements required to obtain a reliable value of  $\phi$ , on the 610 mm diameter bed packed to a depth of 152 mm with 25.4 mm plastic pall rings. The maldistributed feed was radially introduced immediately below the bed through a 100 mm diameter feed pipe at a rate of 24.35 m/sec. A total of 749, 188 and 89 point velocity measurements were made distributed on 20 mm, 40 mm and 60 mm square pitch above the bed respectively.

The results show that 188 point velocity measurements give a reliable estimates of both the maldistribution factor,  $\phi$ , and the total flow. For more confidence, a sample size of 200 measurements was used throughout the work described in this section, distributed on 40 mm square pitch in the prototype and 25 mm square pitch in the model. The configuration of the positions of these points is shown in Fig 6.5.

### 6.4 Experimental Procedure

The two geometrically similar columns, the model and the prototype, were packed with 16 mm and 25.4 mm geometrically similar plastic pall rings respectively to proportional heights so as to



form a series of couples of geometrically similar packed beds, each couple is operated at the same Reynolds number.

This procedure was carried out in both the ideally distributed case and the maldistributed case. Each bed was repacked three times, each time a total of 200 point velocity measurements is taken at fixed positions as shown in Fig. 6.5.

## 6.5 Results

The complete sets of these observations are given in Appendix A3.

The processed data is summarised in Table 6.1 where the maldistribution factor,  $\phi$ , was obtained for the geometrically and dynamically similar beds: Each bed was repacked three times and hence three  $\phi$  values were obtained for each bed.

$\bar{\phi}$ , the average value of  $\phi$  was then calculated, which describes gas maldistribution in that packed bed.

### 6.5.1 Variation of $\bar{\phi}$ with Reynolds Number and Packed Height for Ideally Distributed Beds of Geometric and Dynamic Similarity

Gas maldistribution, expressed by  $\bar{\phi}$ , for dynamically and geometrically similar, ideally distributed packed beds, are listed in Table 6.1. The model bed, ( $D = 381 \text{ mm}$ ), is packed to depths of 63.5 mm and 127 mm, operated at a range of gas Re values. The prototype bed, ( $D = 610 \text{ mm}$ ), is packed to heights of 1.6 times those of the model; i.e. 101 mm and 203 mm. One further 610 mm deep bed was examined.

Variations of  $\bar{\phi}$  for these beds with packed height, P.H., and Re are shown in Graph 6.1, which shows that the value of  $\bar{\phi}$  is

not changed either with P.H. nor with Re. Also,  $\bar{\phi}$  is the same for both beds with a value of about 20.

This means that the minimum value of  $\phi$  observed for a packed bed by using the technique proposed in this work, is around 20, i.e.

$$\phi_{\min} \approx 20$$

#### 6.5.2 Variation of $\bar{\phi}$ with Re and P.H. for Maldistributed Beds of Geometric and Dynamic Similarity

$\bar{\phi}$  values were calculated for the two beds at different heights and gas Re when the feed was badly distributed below the bed.

The 381 mm diameter bed was packed to 63.5 mm, 127 mm and 381 mm heights and was operated at a range of gas Re values.

The 610 mm diameter bed was packed to heights of 101.6 mm, 203 mm and 610 mm and was run under the same range of Re values.

The variation of  $\bar{\phi}$  with Re in both beds is shown in Graphs 6.2 and 6.3 respectively.

The variation of  $\bar{\phi}$  with packed height is given in Graphs 6.4 and 6.5 for the model and the prototype respectively.

#### 6.6 Discussion of the Results

For the ideally distributed bed; (evenly distributed feed entering the bottom of the bed), the obtained value of  $\phi_{\min}$  represents the best distribution that can be achieved in a packed bed, because this distribution could not be improved either by changing the geometry, (the packed depth), nor by changing the operating conditions, (Re values). For this reason,  $\phi_{\min}$  could

be considered as a standard value, such that the goodness of any other distribution in a packed bed can be compared to or measured by that standard.

In the shallow maldistributed packed beds, (maldistributed feed enters the bottom of the bed), the gas maldistribution is severe and thus the values of  $\phi$  are very high, but as the packed depth increases, the values of  $\phi$  decrease, until eventually the value of  $\phi$  obtained for that maldistributed bed converges with  $\phi_{\min}$  which represents ideal conditions. This can be shown clearly from graphs 6.4 and 6.5.

A maldistributed bed with enough packed depth to bring the gas maldistribution to the minimum, will adopt the facility of unresponding to any change in Re value; i.e. it will behave like an ideally distributed bed, this is very clearly shown in Graphs 6.4 and 6.5 where at short packed depths,  $\bar{\phi}$  values vary with Re and this variation is reduced as the bed becomes deeper.

From this, it could be concluded that the packing itself acts as a distributor which improves the distribution of the gas passing through it, but on the other hand there will be many pathways that the gas follows throughout the packing, and hence, the gas is not evenly distributed. This means that the process of improving gas distribution by the packing is limited by the presence of such pathways.

At this stage, a method of testing the quality of feed gas distributors can be developed:

The quality of any gas distributor in any packed column can be tested by using the technique described in this work to measure gas distribution above a very short packed depth, compared to the diameter of the bed, then increasing the packed depth by

intervals, at each interval, the maldistribution factor  $\phi$  is to be measured, until reaching a packed height above which,  $\phi$ , is constant; i.e.  $\phi_{\min}$ . Then the best gas feed distributor is that which produces gas maldistribution closest to the minimum maldistribution at shortest packed depth.

But this method seems very difficult or impossible in a real-life column due to the practical difficulties facing the implementation of this test.

The results also show that if two beds of different sizes designed such that every single dimension in the small size bed (model) is exactly geometrically similar to that in the large-size bed (prototype), are operated at the same value of gas Re, i.e. geometrically and dynamically similar packed beds, then the flow patterns of the gas through these two beds are similar.

This could be seen from Graph 6.6, where the curves show that for a given value of  $P.H/D$  for both beds, there is only one value of  $\bar{\phi}$  at a given Re value, or in other words, the gas maldistribution, or flow pattern, is similar in two geometrically and dynamically similar packed beds.

From that it could be concluded that shallow large diameterpacked beds (or packed beds in general) can be investigated using small models of exact geometric and dynamic similarity to the full-scale packed beds, and the dynamic similarity is achieved by maintaining similar gas Reynolds number.

From these rules of scale-up, the developed method of testing feed gas distributors can be generated to cover the real life gas distributors, by building a model packed bed which is exactly geometrically and dynamically similar to the full-scale column, and then performing laboratory tests on the model.

Apart from these two achievements; (the scale-up rules and the method of testing feed gas distributors), the results show an interesting phenomenon, which is that the obtained values of  $\phi$  are scattered around an average value within a range not more than  $\pm 8\%$ , which is much narrower than that experimentally measured for the 305 mm bed packed with 25.4 mm plastic pall rings which was about  $\pm 30\%$ . This means that as the bed diameter to pack size ratio increases, the values of  $\phi$  become more reproducible.

#### 6.7 Conclusions

From this work, the following conclusions can be made:

- (1) The gas flow patterns are similar in geometrically and dynamically similar randomly packed beds.
- (2) The quality of feed gas distributors can be tested by a method based on the variation of maldistribution factor,  $\phi$ , with the packed height.
- (3) As the packed height to bed diameter ratio increases the values of  $\phi$  decreases until reaching a minimum value,  $\phi_{\min}$ , at a certain depth.
- (4) The observed  $\phi$  values become more reproducible as the bed diameter to pack size ratio increases.

#### 6.8 Summary

The rules of scale-up of large diameter packed beds have been determined experimentally and a method of testing the quality of feed gas distributors based on the variation of gas maldistribution with packed height, and on the determined scale-up rules. The development of this test method and the determination of these rules are originally based on the defined maldistribution factor  $\phi$ . The

meaning of  $\phi$  and its reliability as a measure of the gas  
maldistribution is to be examined in the next section of the  
work.



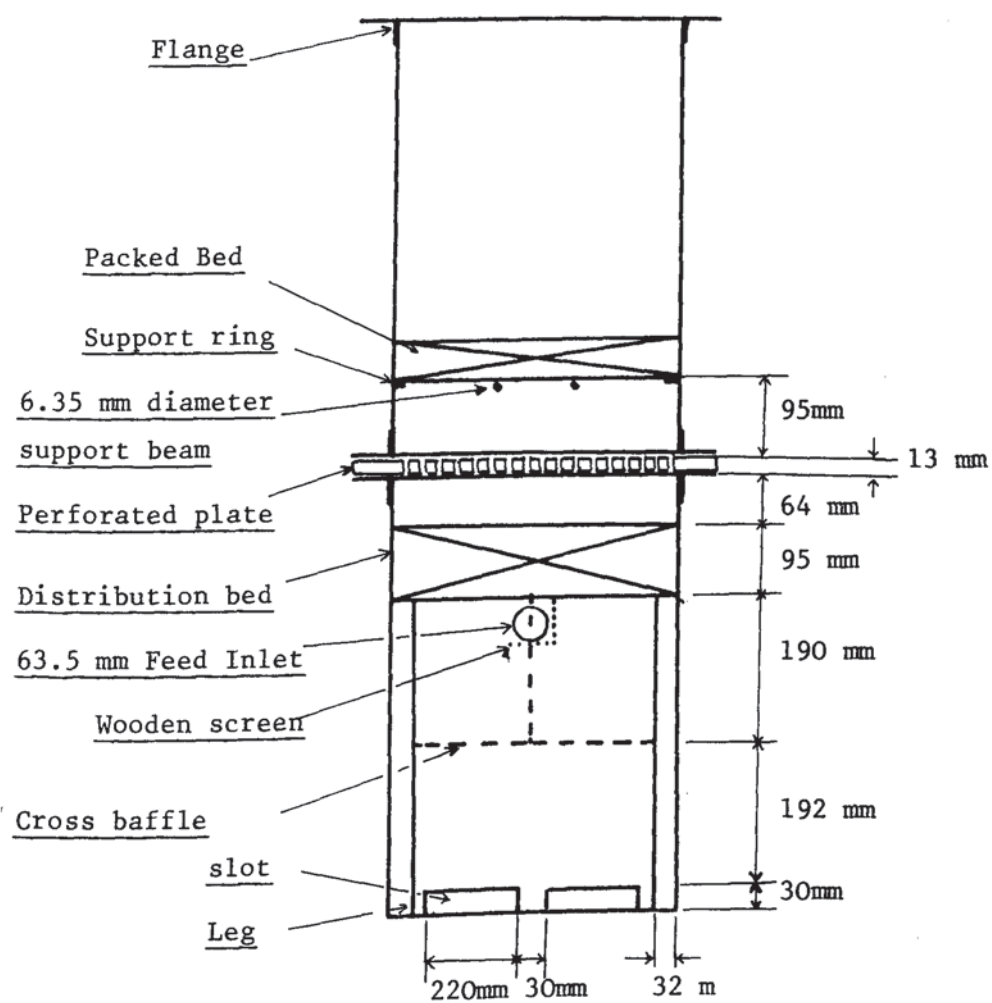


Fig. 6.1 Ideally distributed model packed  
column of 381 mm diameter

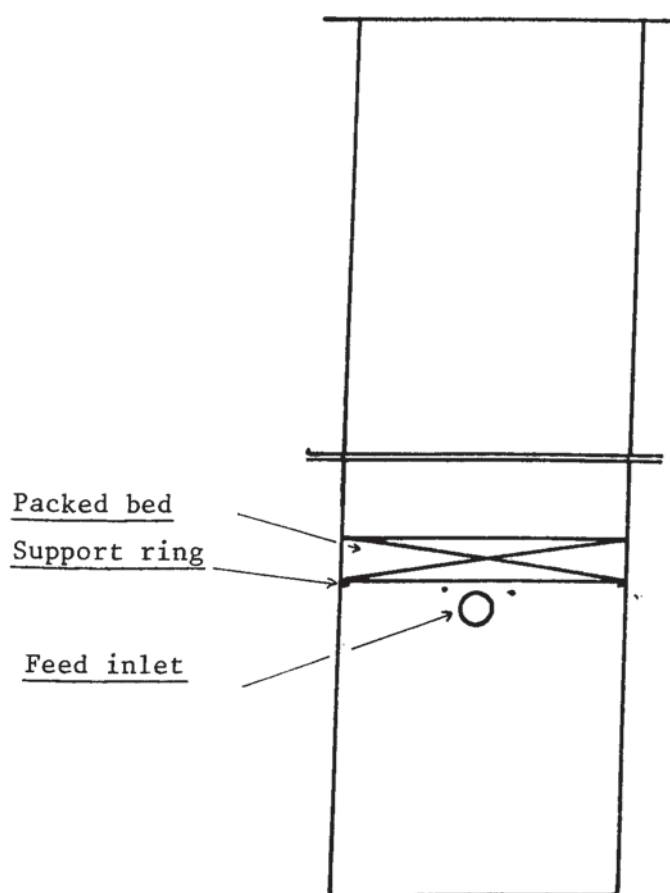


Fig. 6.2 Maldistributed model packed column of 381 mm  
diameter

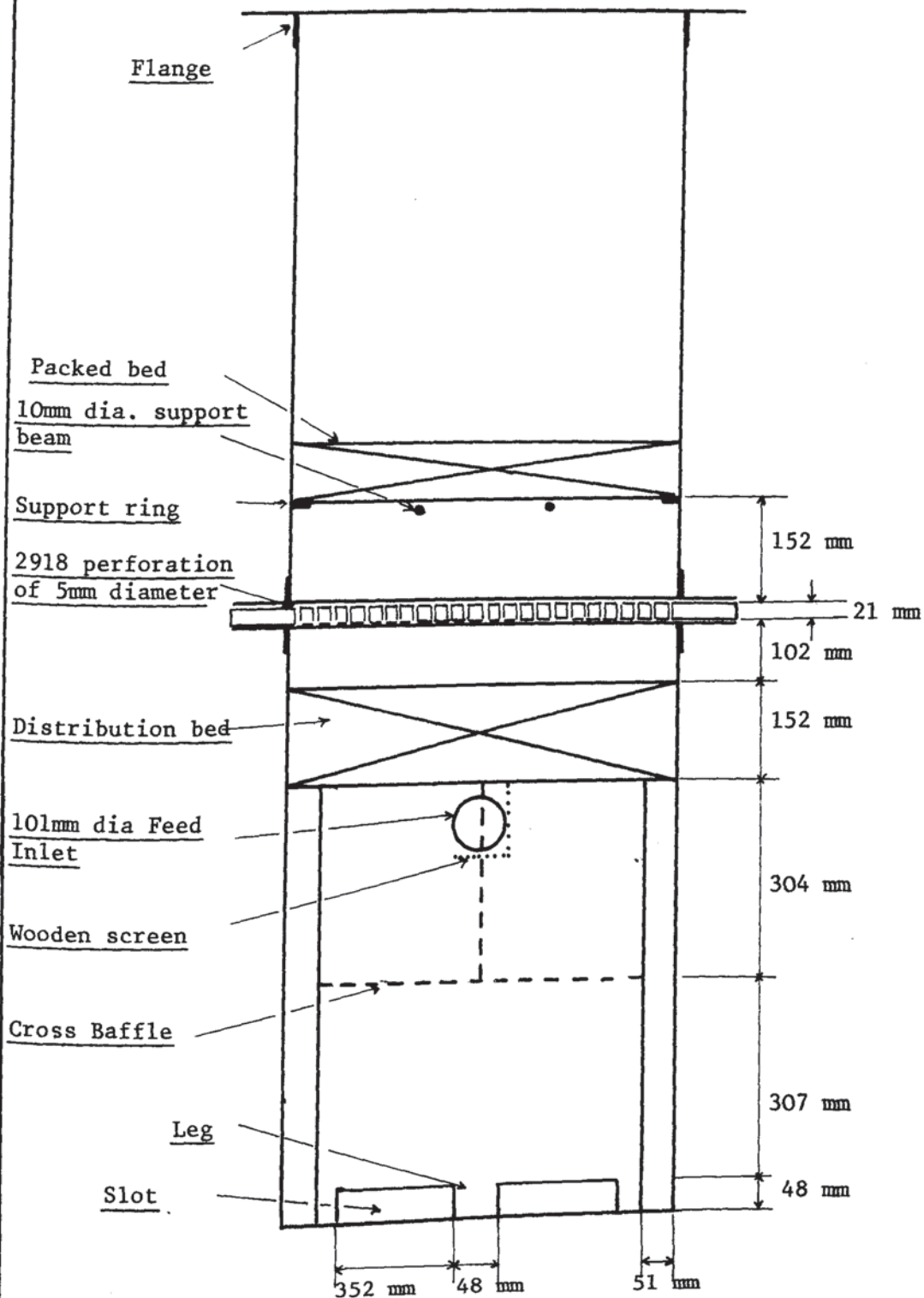


Fig. 6.3 Ideally distributed prototype packed column  
column of 610 mm diameter

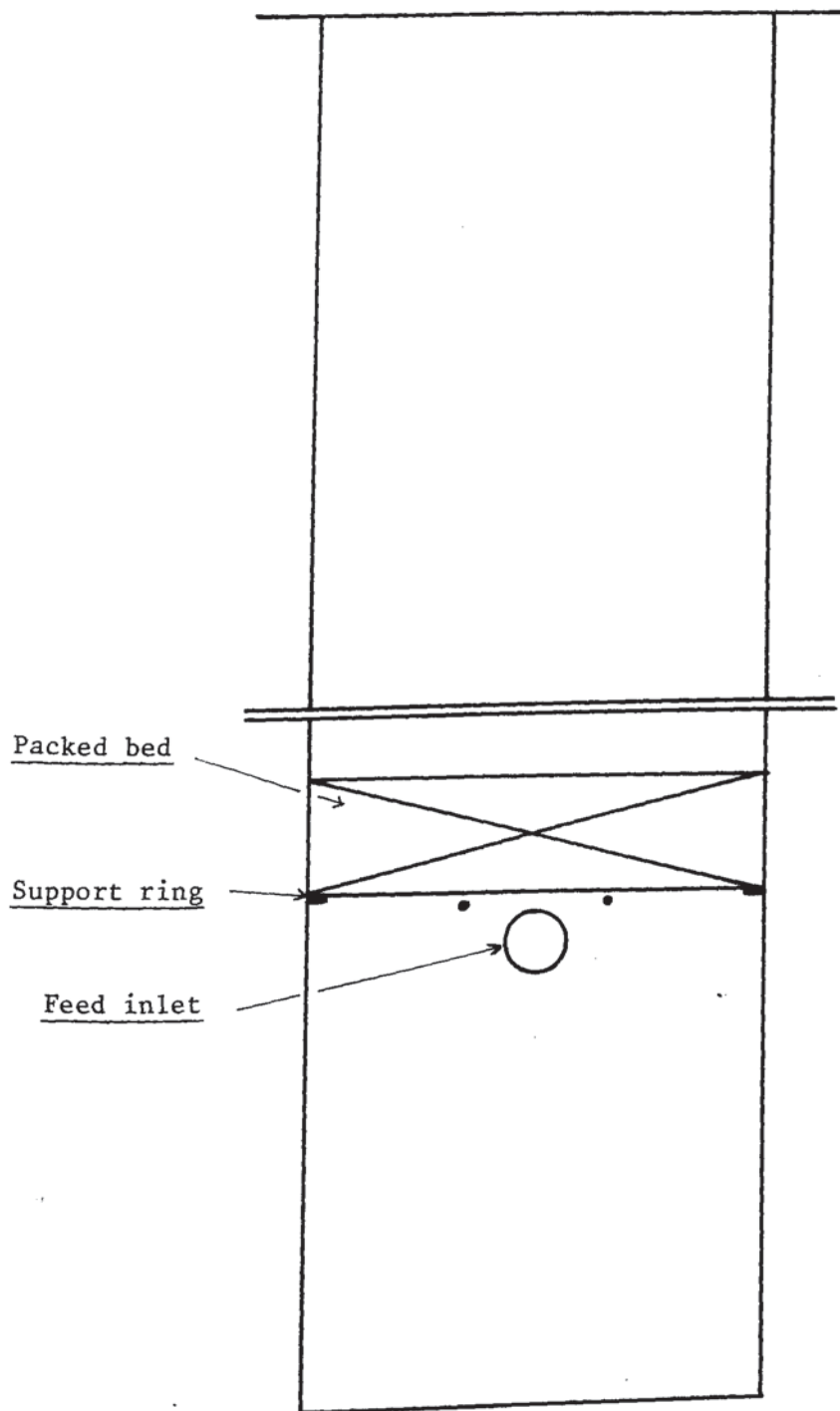


Fig. 6.4 Maldistributed prototype packed column of  
610 mm diameter

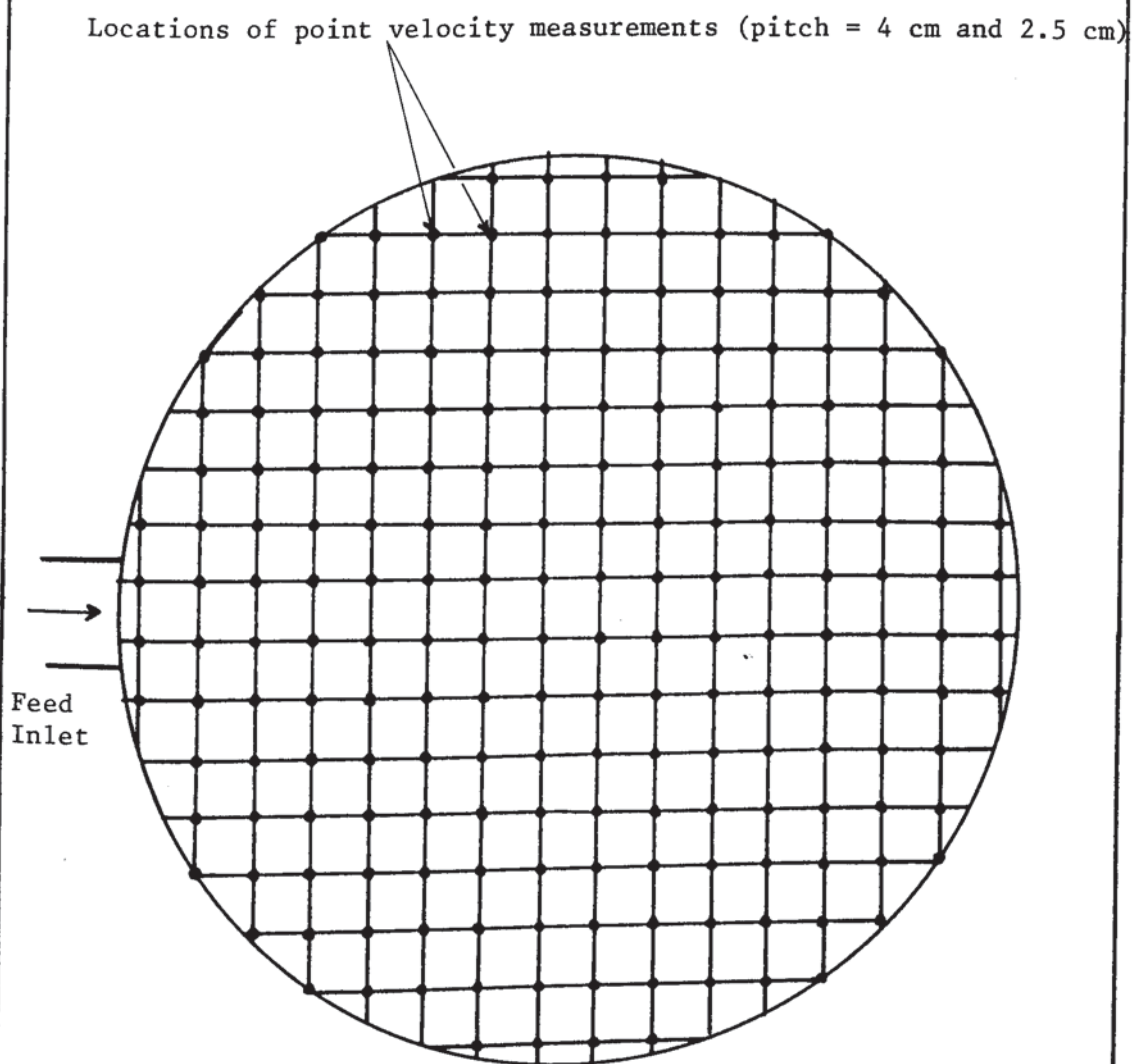


Fig. 6.5 Configuration of the 200 point velocity  
measurements

Table 6.1    Maldistribution Factor,  $\phi$ , for Geometrically and  
Dynamically Similar Packed Beds

NOTE: SEE COMMENT 2 ON P.54 FOR  $\phi$  VALUES.

Status	D (mm)	H (mm)	Re	Repack	$\phi$	$\bar{\phi}$
Maldistributed	381	63.5	18000	1	70.6	75.17
				2	76.8	
				3	78.1	
			25000	1	70.0	73.3
				2	70.9	
				3	79	
			32000	1	81.6	85.1
				2	85.4	
				3	88.4	
	127	18000	18000	1	28.0	29.82
				2	29.76	
				3	31.8	
			25000	1	26.92	28.2
				2	27.61	
				3	30.17	
			32000	1	27.75	29.03
				2	29.2	
				3	30.14	
	381		30000	1	19.1	20.15
				2	20.22	
				3	21.15	
	610	101.6	18000	1	73.2	74
				2	73.8	
				3	74.9	

/Continued .....



Table 6.1 continued

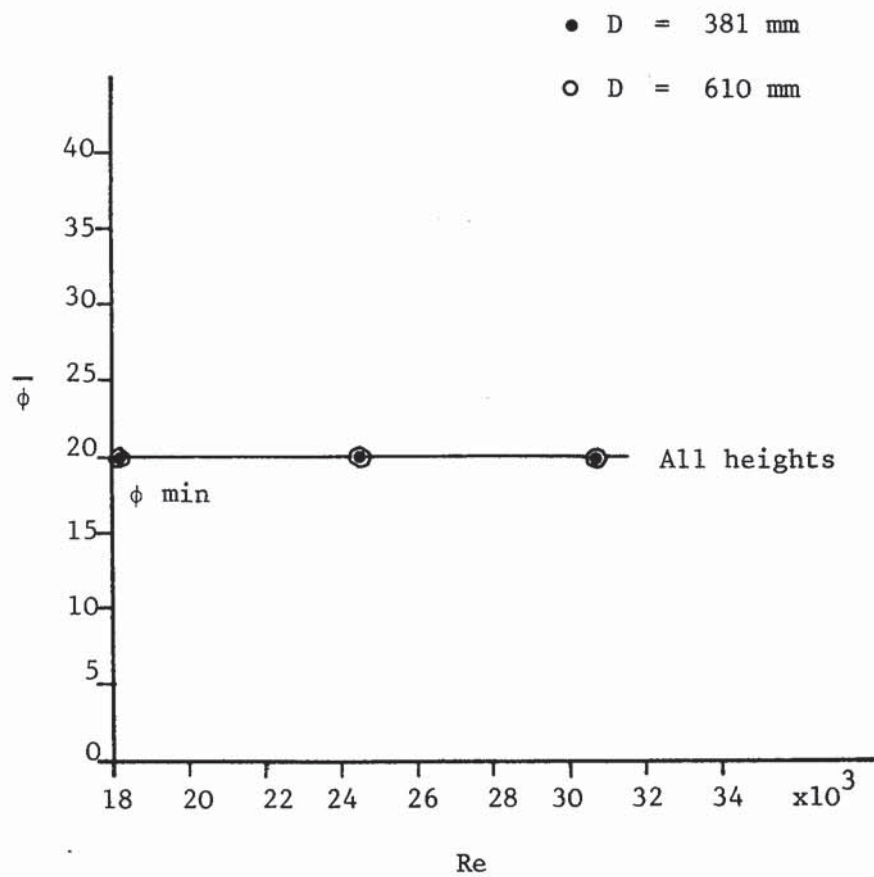
Status	D (mm)	H (mm)	Re	Repack	$\phi$	$\bar{\phi}$
Maldistributed	610	101.6	25000	1 2 3	72 72.16 81.65	75.27
			32000	1 2 3	81.46 81.74 92.15	85.11
		203	18000	1 2 3	27 27.5 30.6	28.4
			25000	1 2 3	26.7 29.3 31.4	29.13
			32000	1 2 3	27.01 29.79 31.08	29.3
	381	63.5	16150	1 2 3	19.45 20.4 21.64	20.4
			24500	1 2 3	19.5 19.56 22.48	20.5
			30700	1 2 3	18.42 20.6 22.43	20.48
		127	16150	1 2 3	19.65 19.9 21.84	20.46
			24500	1 2 3	19.13 20.2 21.96	20.43

/Continued .....

Table 6.1 continued

Status	D (mm)	H (mm)	Re	Repack	Ø	Ø̄
Ideal Distribution	381	127	30650	1	19.65	20.88
				2	20.55	
				3	22.45	
	610	101.6	20000	1	19.63	20.28
				2	20.28	
				3	20.93	
			26000	1	19.72	20.1
				2	19.95	
				3	20.64	
			29000	1	19.73	20.72
				2	20.13	
				3	22.3	
		203	20000	1	19.7	20.6
				2	19.95	
				3	22.14	
			26000	1	20.1	20.3
				2	20.13	
				3	20.67	
			29000	1	19.3	20.37
				2	20.5	
				3	21.3	
		610	30000	1	20.0	20.2
				2	20.18	
				3	20.47	

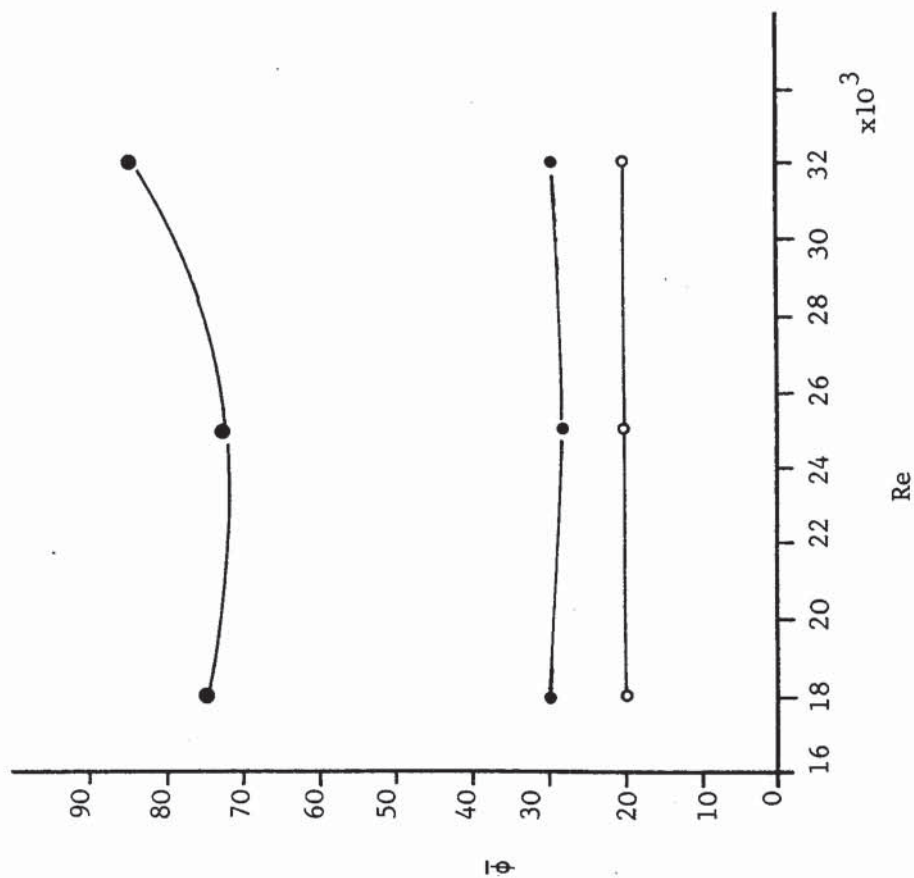
Graph 6.1    $\bar{\phi}$  against Re for Two Geometrically  
similar Beds of Different Heights  
at Ideal Distribution Situation



P.H. (mm) = ● 381

● 127

○ 63.5



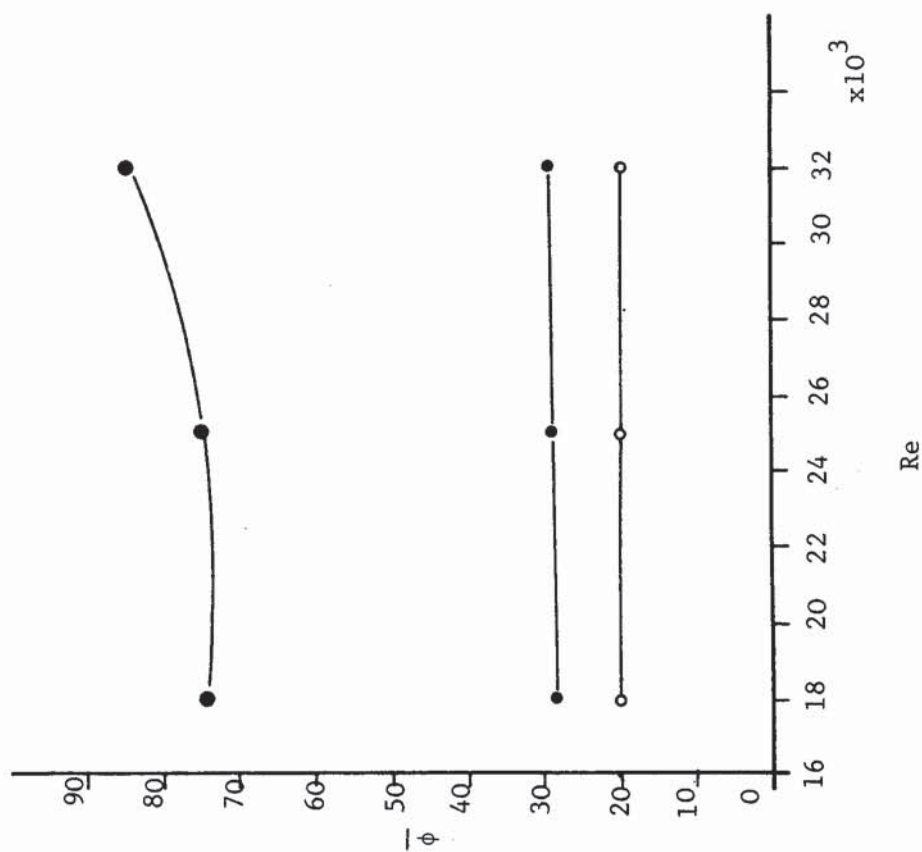
Graph 6.2 Variation of  $\bar{\phi}$  with Re in a 381 mm

diameter maldistributed bed

P.H. (mm) = ● 610

● 203

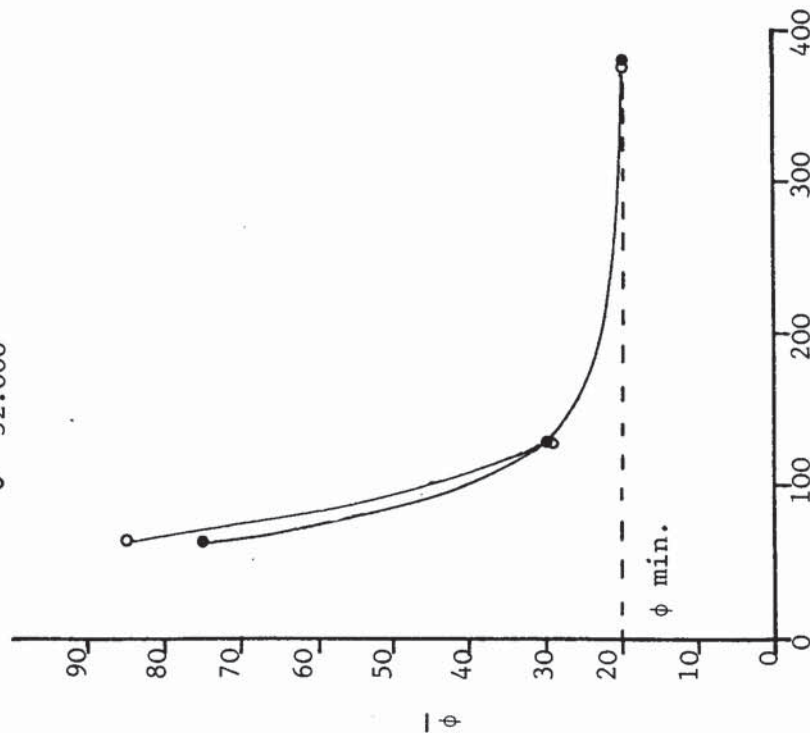
○ 101.6



Graph 6.3 Variation of  $\bar{\phi}$  with Re in a 610 mm

diameter maldistributed bed

Re  
● 18.000  
○ 32.000

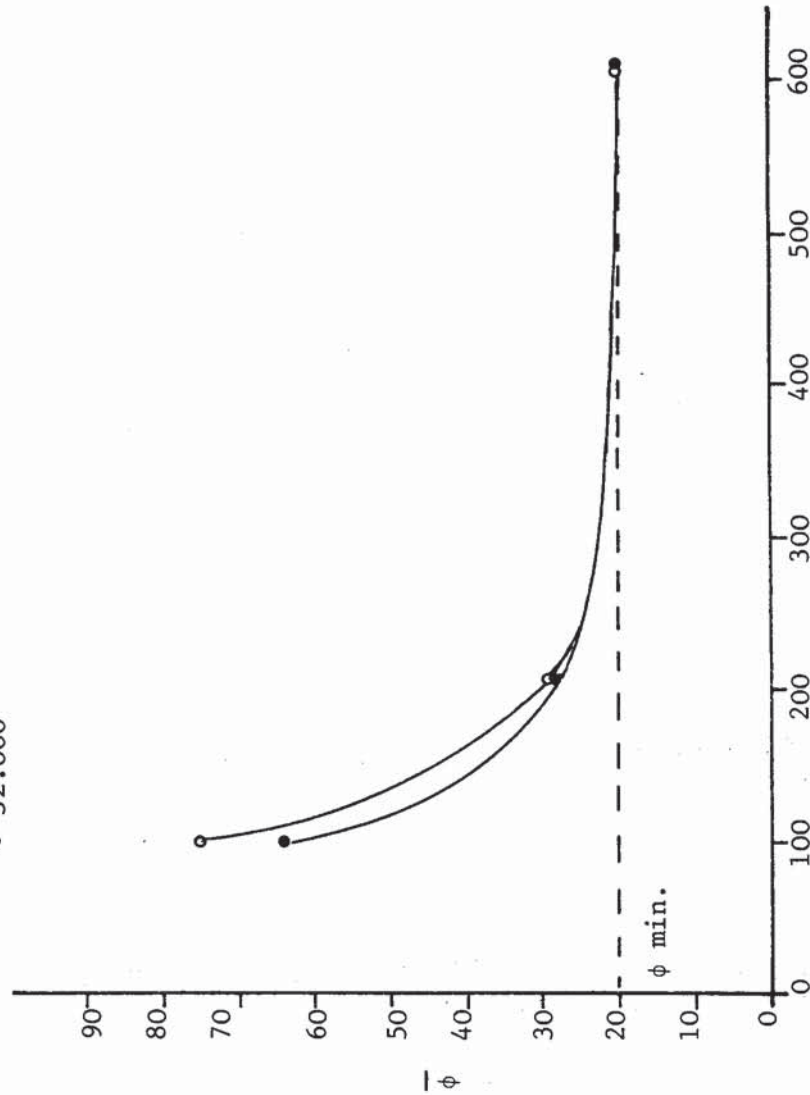


P.H. (mm)

Graph 6.4 Variation of  $\phi$  with P.H. in the

381 mm diameter maldistributed bed

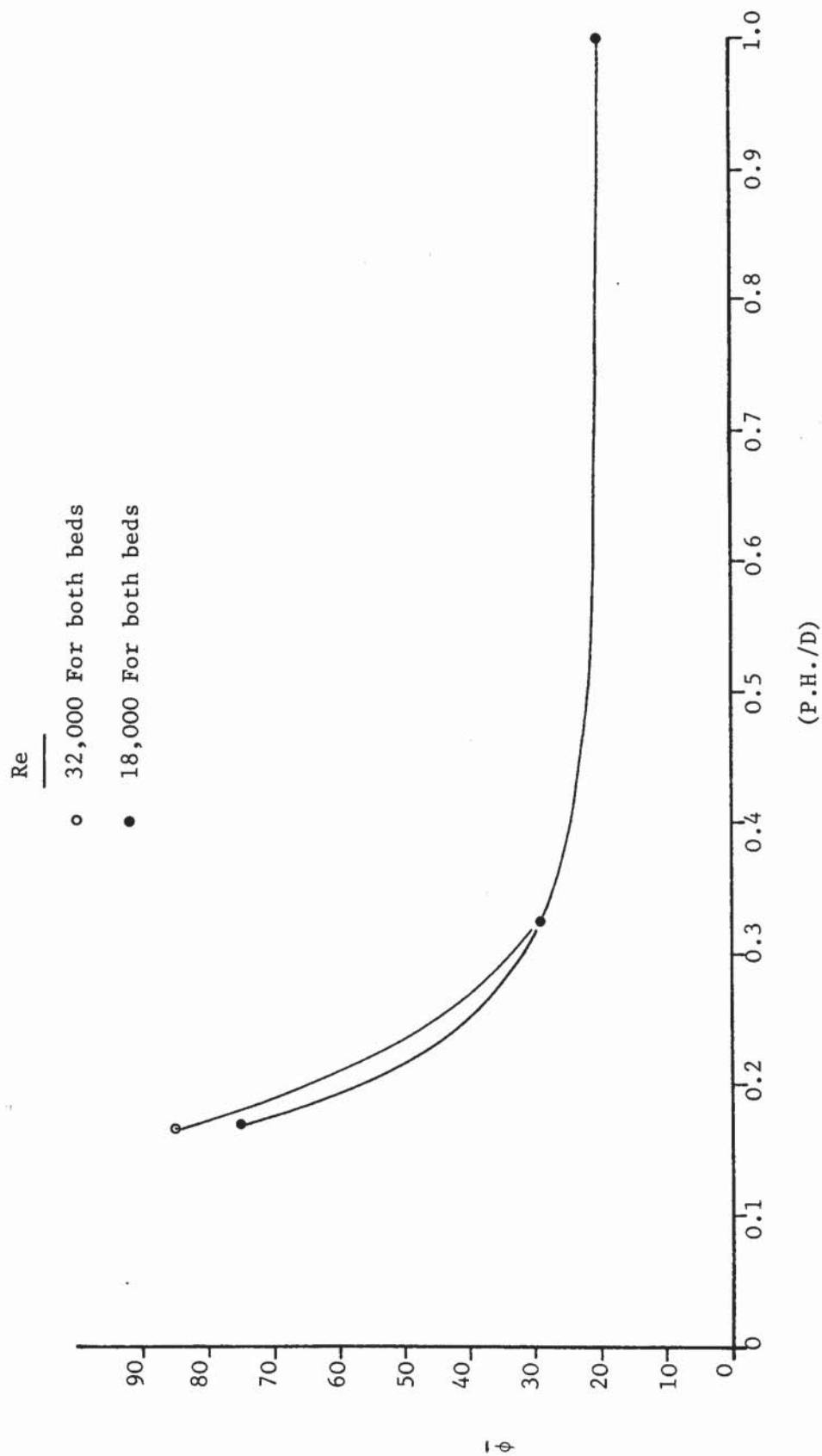
Re  
● 18.000  
○ 32.000



P.H. (mm)

Graph 6.5 Variation of  $\phi$  with P.H. in the 610 mm diameter

maldistributed bed



Graph 6.6 Variation of  $\bar{\phi}$  with (P.H./D) for geometrically and dynamically similar maldistributed packed beds



SECTION VII  
STATISTICAL AND GRAPHICAL INTERPRETATION OF THE  
EXPERIMENTAL RESULTS

Introduction

In this work, the experimental results are statistically interpreted by Multi-Factor Analysis of the Variance, and graphically illustrated by producing Response Surfaces to visualise the profile of the gas emerging from the top of a packed bed.

It was necessary to define the maldistribution factor  $\phi$ , which is proportional to the variance, because of the random nature of packed beds. Noting that statistics is the study of random variations, in this section the nature of  $\phi$  is studied and the validity of the scale-up rules is proved by showing that there is no statistically significant effect of column size upon the gas flow pattern for geometrically similar beds operated at the same Reynolds number.

7.1      Analysis of the Distribution of Gases Emerging from the  
Top of Packed Beds

In this part of the work an attempt has been made to investigate the effect of the spatial coordinates  $x$  and  $y$  on the gas distribution emerging from the top of packed beds by the use of Two-Factor analysis of the variance, whose mathematical formulation is given in Appendix A4, to show that:

(a) For a well-distributed gas,  $\phi = \phi_{\min}$ , there is no spatial maldistribution, i.e. the effect of  $x$  and  $y$  is statistically insignificant.

(b) For a maldistributed gas, the effect of position, i.e. x and y spatial coordinates is significant and this is associated with  $\phi > \phi_{\min}$ .

The two-factor analysis of the variance produces values, (computed f-values), to determine the effects of:

- (1) x-spatial coordinate, (Factor-X).
- (2) y-spatial coordinate, (Factor-Y).
- (3) Interaction of Factor-X with Factor-Y.
- (4) Repacking the bed.

on the distribution of air emerging from the top of a randomly packed bed.

These computed f-values are to be compared with their corresponding standard values found in the statistical textbooks, (tabulated F-values), at a predetermined level of confidence ( $\alpha$ ).

If the computed f-value for any factor is greater than or equal to its corresponding tabulated f-value, then that factor has a significant effect on the distribution and the gas is maldistributed due to that factor; if the computed f-value is less than the tabulated f-value, then that factor has no significant effect on the gas distribution.

The most important factors in this analysis are the x and y-factors whose computed f-values determine the gas maldistribution in the x and y directions.

If both, or one of them is significantly effective, then this means that the gas is severely maldistributed along both or one of the x and y-axes in the xy-plane above the bed. The x-axis represents the direction at which the gas feed is introduced to the column below the bed, (radial feed inlet). The y-axis is at right angles to the feed direction in xy-plane. The relationship

between  $\phi$  and the computed f-values is discussed in detail at a later stage of this section.

#### 7.1.1 Method

A sample of 200 point velocity measurements was used to evaluate the maldistribution factor  $\phi$ . The locations of these points are equally distributed on a circular plane, see Fig. 6.5. But in order to fulfill the requirement of the two-factor analysis of the variance, (that is each position in x-direction should be used once in conjunction with each position in y-direction, or in other words, all the positions of point velocities should be specified by an x-coordinate and y-coordinate, see Appendix A4), and to use the largest possible sample size, (number of points), a sample size of 144 point velocity measurements, equally distributed in a square plane which represent the central part of the circular plane, was used in this analysis, see Fig. 7.1.

#### 7.1.2 Results of the Gas Distribution Analysis and Discussion

The two-factor analysis of the variance has been performed on the observed point velocities above some of the packed beds described in Section 6 to study the effect of packed height on the gas distribution in both ideally and badly distributed feed situations.

Two sample solutions, one for a maldistributed bed and the other for an ideally distributed bed, are given in Appendix A4.

The results are summarized in Table 7.1.

The computed f-values obtained for any bed are to be compared with the standard tabulated f-values listed below

Table 7.1. For example, for the ideally distributed bed, 610 mm diameter and 203 mm height, the computed f-value for the effect of repacking the bed is 0.62. This value is to be compared with the standard f-value at 0.05 level of confidence ( $\alpha$ ) for the repacking effect, which is 3; the computed f-value is less than standard f-value, therefore, repacking the bed has no significant effect on the total picture of the distribution of the gas above that bed. Similarly, the computed f-value of x-effect =  $1.46 < f_{0.05} = 1.79$ , which means that the gas is not significantly maldistributed in the x-direction.

On the other hand, for the 610 mm diameter and 101.6 mm packed depth maldistributed bed, the computed f-value of x-effect =  $47.06 > f_{0.05} = 1.79$ , which means that the gas is severely maldistributed in the direction of the feed, (x-direction).

The results show that the computed f-values for the ideally distributed bed, 610 mm diameter and P.H. of 101.6 mm, are nearly equal to their corresponding values obtained for the same bed when its packed depth was doubled and, also, similar to those obtained for the ideally distributed 381 mm diameter bed of different Re value. This means that neither the packed height nor the Re values has any effect on gas flow patterns in ideally distributed packed beds. The computed f-values for the x-effect and y-effect show some maldistribution, however not significant, in both x and y direction; if the gas was evenly distributed then the computed f-values would have been equal to zero. This shows that there is a minimum maldistribution resulting from the point to point velocity variations.

For the geometrically and dynamically similar prototype and model beds packed to 101.6 mm and 63.5 mm respectively, the

computed f-values show severe gas maldistribution in the x-direction and slightly significant maldistribution in the y-direction. The similar magnitudes of the computed f-values show the similarity in maldistribution in both beds both in the x-direction and y-direction. As the P.H./D ratio increases in these similar beds, the computed f-values of the x-effect decreases to show that the maldistribution in the x-direction has been reduced to a new equal level in both beds, and becomes insignificant in the y-direction. At a deeper bed, the computed f-values show insignificant maldistribution in both x-direction and y-direction.

This shows that gas flow patterns are similar in maldistributed geometrically and dynamically similar beds, and that, gas maldistribution decreases as the packed height increases.

From the two-factor analysis of the variance, it could be concluded that gas flow patterns are similar in geometrically and dynamically similar packed beds, which is the same conclusion abstracted from using the simply defined maldistribution factor  $\phi$  in the previous section of this work.

In the following part of this section, the similarity of flow patterns in geometric and dynamic similar beds is to be studied in more detail.

## 7.2 Analysis of the Gas Distribution in Geometric and Dynamic Similar Packed Beds

The distribution of the gas emerging from the top of geometric and dynamic similar beds of different size is investigated statistically by the use of Three-Factor Analysis of the Variance to show that at dynamic similarity, changing the scale of operation has an insignificant effect on the flow pattern. The mathematical



formulation of this analysis is given in Appendix A4.

Similar to the two-factor analysis of the variance, the three-factor analysis of the variance produces computed f-values to estimate the effect of the spatial coordinates x and y as well as the effect of the packed bed diameter on the flow patterns. These computed f-values are to be compared to the corresponding tabulated f-values at predetermined significance level,  $\alpha$ .

#### 7.2.1 Method

A sample size of 144 point velocity measurements, similar to that shown in Fig. 7.1, is used in this analysis. But instead of the gas velocity, the Reynolds numbers at these points are used, where  $Re$  is defined by the point velocity and the column diameter. These  $Re$  values are subsequently called as "point Reynolds numbers".

The point  $Re$ s obtained for a bed repacked several times (3 times) are used in conjunction to other point  $Re$ s obtained for another bed, of geometric and dynamic similarity to the first bed but of different size, repacked three times.

#### 7.2.2 Results and Discussion

The distribution of the gas emerging from the top of two geometrically and dynamically similar beds is analysed in both situations of the ideally distributed feed and the badly distributed feed. The results are summarised in Table 7.2.

Comparison of the computed f-values with their corresponding tabulated f-values at a  $\alpha = 0.05$ , listed in Table 7.2, shows that the diameters of the two beds have no significant effect on the distribution of the gas in both ideally and badly distributed



feed situations. In the ideally distributed beds,  $\phi = \phi_{\min}$ , gas maldistribution is insignificant in the x and y spatial direction, in maldistributed beds, associated with  $\phi > \phi_{\min}$ , the gas is severely maldistributed in x-direction and slightly maldistributed in y-direction.

From this it could be concluded that gas flow patterns are similar in geometrically and dynamically similar packed beds and this supports the same result concluded by using  $\phi$  to establish the rules of packed beds scale-up.

### 7.3 A Discussion of the Definition of $\phi_{\min}$ in Terms of Multi-Factor Analysis of the Variance

In this part of the work an attempt is made to highlight the relationship between the estimates of the variance computed by the use of the multi-factor analysis of the variance technique and the value of  $\phi$  obtained to describe the quality of the gas distribution above a packed bed. This relationship was shown clearly in the work performed above in this section, where for a well-distributed bed,  $\phi$ , has the minimum value,  $\phi_{\min}$  and the multi-factor analysis of the variance shows that the maldistribution of the gas is insignificant; for a maldistributed packed bed,  $\phi > \phi_{\min}$  and the maldistribution is significant.

$\phi$  is defined by Eqn. 5.3 as:

$$\phi = \frac{n}{\sum_{i=1}^n} \left(1 - \frac{V_i}{\bar{V}}\right)^2$$

∴ From Eqn. 5.2

$$\sigma^2 = \frac{\bar{V}^2}{n-1} \phi \quad \dots\dots (7.1)$$

The five estimates of the variance,  $\sigma^2$ , obtained by the two-variable analysis of the variance, see Appendix A4, are:

$$E(S_1^2) = E\left[\frac{SSx}{a-1}\right] = \sigma^2 + \frac{rb \sum_{i=1}^a \alpha_i^2}{a-1} \quad \dots\dots (7.2a)$$

$$E(S_2^2) = E\left[\frac{SSy}{b-1}\right] = \sigma^2 + \frac{ra \sum_{j=1}^b \beta_j^2}{b-1} \quad \dots\dots (7.2b)$$

$$E(S_3^2) = E\left[\frac{SS(xy)}{(a-1)(b-1)}\right] = \sigma^2 + \frac{r \sum_{i=1}^a \sum_{j=1}^b (\alpha\beta)_{ij}^2}{(a-1)(b-1)} \quad \dots\dots (7.2c)$$

$$E(S_4^2) = E\left[\frac{SSR}{(r-1)}\right] = \sigma^2 + \frac{ab \sum_{k=1}^r \rho_k^2}{r-1} \quad \dots\dots (7.2d)$$

$$E(S^2) = E\left[\frac{SSE}{(ab-1)(r-1)}\right] = \sigma^2 \quad \dots\dots (7.2e)$$

where:  $S_1^2$ ,  $S_2^2$ ,  $S_3^2$ ,  $S_4^2$  and  $S^2$  are the factor x, factor y, interaction, repacking and error mean squares respectively.

$SSx$ ,  $SSy$ ,  $SSxy$ ,  $SSR$ ,  $SSE$  are the factor x, factor y, interaction, repacking and error sum of squares respectively.

- a: Is the number of point velocity measurements in x-direction, (a=12).
- b: Is the number of point velocity measurements in y-direction, (b=12).
- r: Is the number of times that the bed is repacked, (r=3).
- $\alpha_i$ : Is the effect of (ith) point velocity in x-direction on the gas distribution.
- $\beta_j$ : Is the effect of (jth) point velocity in y-direction on the gas distribution.
- $(\alpha\beta)_{ij}$ : Is the interaction, or joint, effect of (ith) point velocity in x-direction and the (jth) point velocity in y-direction on the gas distribution
- $\rho_k$ : Is the effect of the (kth) replicate.

For an ideally distributed feed below a packed bed, the maldistribution of the gas above that bed is described by  $\phi_{\min}$  which means that:

$$\sigma^2 = \frac{\bar{V}^2}{n-1} \phi_{\min} \quad \dots (7.3)$$

It is noticed in Table 7.1 that the interaction (xy) and the repacking effects are insignificant in both cases of ideally and badly distributed feeds. Thus in the ideally distributed feed case, (well distributed bed), it can be said that the total effect of the x-direction is the minimum.

$$\text{i.e. } \sum_{i=1}^a \alpha_i^2 = \left( \sum_{i=1}^a \alpha_i^2 \right)_{\min} \quad \dots (7.4)$$

And that the total effect of the y-direction on the gas distribution is the minimum, i.e.

$$\sum_{j=1}^b \beta_j^2 = \left( \sum_{j=i}^b \beta_j^2 \right)_{\min} \quad \dots (7.5)$$

Substituting Eqns. 7.3, 7.4 and 7.5 in Eqns. 7.2a-7.2e will produce five estimates of  $\phi_{\min}$  as follows:

$$E \left[ \frac{SSx}{a-1} \right] = \frac{\bar{V}^2}{n-1} \phi_{\min} + \left[ \frac{rb \sum_{i=1}^a \alpha_i^2}{a-1} \right]_{\min} \dots (7.6a)$$

$$E \left[ \frac{SSy}{b-1} \right] = \frac{\bar{V}^2}{n-1} \phi_{\min} + \left[ \frac{rb \sum_{j=1}^b \beta_j^2}{b-1} \right]_{\min} \dots (7.6b)$$

$$E \left[ \frac{SS(xy)}{(a-1)(b-1)} \right] = \frac{\bar{V}^2}{n-1} \phi_{\min} + \frac{\sum_{i=1}^a \sum_{j=1}^b (\alpha\beta)_{ij}^2}{(a-1)(b-1)} \dots (7.6c)$$

$$E \left[ \frac{SSR}{(r-1)} \right] = \frac{\bar{V}^2}{n-1} \phi_{\min} + \frac{ab \sum_{k=1}^r \rho_k^2}{r-1} \dots (7.6d)$$

$$E \left[ \frac{SSE}{(ab-1)(r-1)} \right] = \frac{\bar{V}^2}{n-1} \phi_{\min} \dots (7.6e)$$

That means that the sum of squares are functions of  $\phi_{\min}$  from which it would be immediately concluded that all five estimates of  $\phi_{\min}$  are unbiased when

$$\sum \alpha_i = \sum \rho_j = \sum (\alpha\beta)_{ij} = \sum \rho_k = 0 \quad \dots (7.7)$$

Eqn. 7.7 is satisfied only if the gas is evenly distributed, and this condition could not be found in the packed beds due to their random nature.

In a maldistributed packed bed  $\phi > \phi_{\min}$  and the gas is maldistributed in the x and y directions, i.e.

$$\sum_{i=1}^a \alpha_i^2 > \left( \sum_{i=1}^a \alpha_i^2 \right)_{\min}$$

and

$$\sum_{j=1}^b \beta_j^2 > \left( \sum_{j=1}^b \beta_j^2 \right)_{\min}$$

and the five estimates of  $\phi$  will be:

$$E \left[ \frac{SSx}{(a-1)} \right] = \frac{\bar{V}^2}{n-1} \phi + \frac{rb \sum_{i=1}^a \alpha_i^2}{(a-1)} \dots\dots (7.8a)$$

$$E \left[ \frac{SSy}{(b-1)} \right] = \frac{\bar{V}^2}{n-1} \phi + \frac{ra \sum_{j=1}^b \beta_j^2}{(b-1)} \dots\dots (7.8b)$$

$$E \left[ \frac{SS(xy)}{(a-1)(b-1)} \right] = \frac{\bar{V}^2}{n-1} \phi + \frac{r \sum_{i=1}^a \sum_{j=1}^b (\alpha\beta)_{ij}^2}{(a-1)(b-1)} \dots\dots (7.8c)$$

$$E \left[ \frac{SSR}{(r-1)} \right] = \frac{\bar{V}^2}{n-1} \phi + \frac{ab \sum_{k=1}^r \rho_k^2}{(r-1)} \dots\dots (7.8d)$$

$$E \left[ \frac{SSE}{(ab-1)(r-1)} \right] = \frac{\bar{V}^2}{n-1} \phi \dots\dots (7.8e)$$

Where the right hand terms represent the effects of the point to point velocity variation and the regional velocity variation, and  $\phi > \phi_{\min}$ .

Fig. 7.1   Configuration of the 144 Points, (bounded by  
dotted line).   Used in Two-Factor Analysis  
of the Variance and Response Surfaces

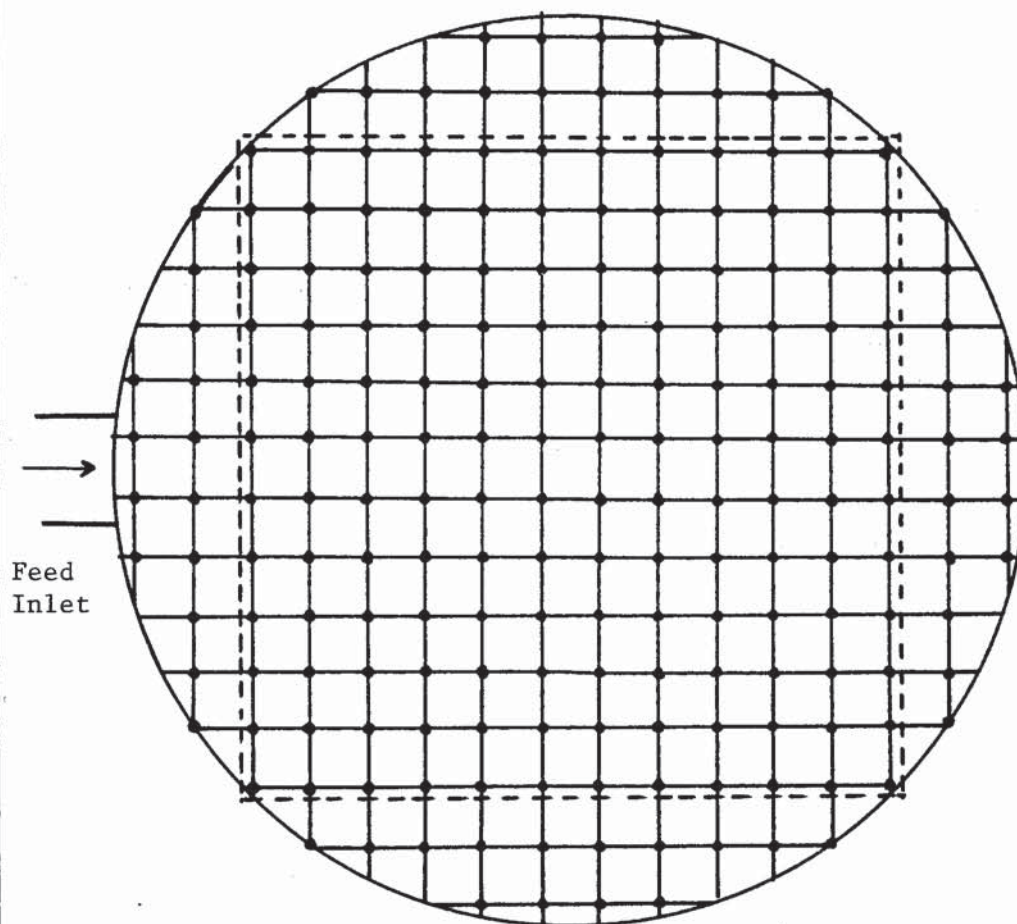




Table 7.1    Computed f-values Result from Two-Factor Analysis  
of the Variance

Status	D (mm)	$\frac{D}{H}$	Re	$\bar{\phi}$	Computed f-values			
					Rep.	x	y	xy
Ideal distribution	610	6	29000	20.72	0.65	1.46	1.18	1.06
	610	3	29000	20.37	0.62	1.405	1.0	1.12
	381	6	30700	20.48	0.611	1.446	0.979	1.02
Maldistributed	610	6	32000	85.11	0.614	47.06	1.823	0.736
	610	3	32000	29.3	0.57	11.8	1.012	1.04
	381	6	32000	85.1	0.587	46.78	2.03	0.853
	381	3	32000	29.03	0.236	11.57	1.13	0.577
	381	1	30000	20.15	0.53	1.43	1.05	1.17

Note: Standard tabulated f-values at 0.05 level of confidence obtained from Appendix A4, Table A4.4 are as follows:

- (1) For x-effect and y-effect:  $f_{0.05} = 1.79$
- (2) For xy effect:  $f_{0.05} = 1.22$
- (3) For repacking effect:  $f_{0.05} = 3$

Table 7.2    Computed f-values Result from Three Factor Analysis  
of the Variance for Geometrically Similar Beds  
Operated at Re = 30000 and D/P.H. = 3

Source of variation	Computed f-values for geometrically similar bed		$f_{0.05}$
	Ideal	Maldistributed	
x	1.613	11.753	1.71
y	1.315	2.216	1.71
D	0.543	0.568	3.84

#### 7.4      Graphical Illustration of the Flow Pattern of the Gas Emerging from the Top of the Packed Beds

In two-factor analysis of the variance, a study was made to estimate the details of the distribution of the gas emerging from the top of packed beds.

In this section of the work the gas maldistribution is going to be studied graphically by producing graphical figures which represent the actual gas velocity distribution above the packed bed. These graphical figures (response surfaces), are formed by drawing lines through the observed point velocities by the use of the computer program listed in Appendix A5. Thus these response surfaces will provide more insight on the idea of the maldistribution of the gas in packed beds as a whole.

##### 7.4.1      Method

The 144 point velocity measurements used previously in two-factor analysis of the variance and shown in Fig. 7.1 are again used here by feeding these observations, for any packed bed dealt with in this work, into the computer program listed in Appendix A5. Each point velocity of these 144 points is the average value of air velocity measured at a point each time the bed is repacked, (3 times). This means that each response surface represents the average gas maldistribution above that packed bed. Two views are produced for each response surface.

##### 7.4.2      Results and Discussion

Response surfaces are produced to show the gas distribution above geometrically and dynamically similar ideally distributed and maldistributed packed beds and presented in Figures 7.2-7.9.

Figures 7.2 and 7.3 show the response surfaces of the gas distribution above two geometrically and dynamically similar ideally distributed packed beds of packed height to diameter ratio of 0.167. The interesting feature of these figures is that each response surface consists of points which go above and below an average value all over the surface. Similarly, Figures 7.4 and 7.5, which show the response surfaces for the same beds above but of packed depth to bed diameter ratio of 0.333, show the same phenomenon. All these response surfaces show a great deal of similarity amongst themselves which indicates a similar gas flow pattern in geometrically and dynamically similar beds of ideal feed distribution.

Figures 7.6 and 7.7 also show a great deal of similarity. They represent a gas distribution above maldistributed beds of P.H./D ratio of 0.167. Although they are not smooth, due to the point to point velocity variation but the most important feature of them is that the existence of areas near the feed inlet consist of point velocities lower than the average velocity, and that part of the surface as a whole is lower than the average value. At the same time, the other part of the surface is much higher than the average value. This means that the maldistribution of the gas is due to two sources. First, the point to point velocity variation which occurs all over the surface and second, the existence of regions of very high velocities and others of very low velocities, this phenomenon is subsequently referred to as regional velocity variation.

As the packed depth increases proportionally in the two beds mentioned above, the regional velocity variation decreases as shown in Figures 7.8 and 7.9 which are highly similar to each other.

From all these figures, the similarity of flow pattern in geometrically and dynamically similar packed beds can be visualized clearly through the similarity between their response surfaces.

#### 7.4.3 Discussion of $\phi$ in Relation to Response Surfaces

Response surfaces produced for an ideally distributed bed show only point to point velocity variations, which are the same beds whose gas maldistributions are described by  $\phi_{\min}$  in Section 6.5.1. At that stage, it was claimed that  $\phi_{\min}$  cannot be equalled to zero due to the existence of point to point velocity variation. The response surfaces demonstrated the existence of such point to point velocity variation.

Also, the similarity between these response surfaces shows the similarity of the flow pattern of gas through these beds. This similarity exists between response surfaces of gas distribution through maldistributed beds of geometric and dynamic similarity which explains the similarity of the obtained  $\phi$  values for such beds and gives some insight on the sources of the gas maldistribution; point to point velocity variations and regional velocity variations which are combined together to produce high  $\phi$  value.

Therefore, the technique of producing response surfaces has provided further proof of the validity of  $\phi$  as a reliable measure of the gas maldistribution through packed beds and also provides more evidence on the validity of the scale-up rules.

#### 7.5 Conclusions

From these two techniques; Multi-factor analysis of the variance and response surface, it could be concluded that:

(1) The maldistribution factor,  $\phi$ , is a reliable measure of the gas maldistribution in packed beds and specially in shallow large diameter packed beds due to its sensitivity to any change in feed distribution; where it has a minimum value for an ideally distributed feed and a much larger value for a maldistributed feed entering the bottom of the bed.

(2) The validity of the scale-up rules has been proved by two more techniques in addition to  $\phi$  technique.

#### 7.6 Summary

In this work, two techniques, Multi-factor analysis of the variance and the response surface, have been used to prove the validity of the scale-up rules and the efficiency of  $\phi$  in measuring gas flow pattern in packed beds. In the next section of the work, an experimental investigation of the fluid mechanics of a maldistributed gas flow, is carried out to know the reasons behind such maldistributions and to provide a better understanding of this phenomenon.



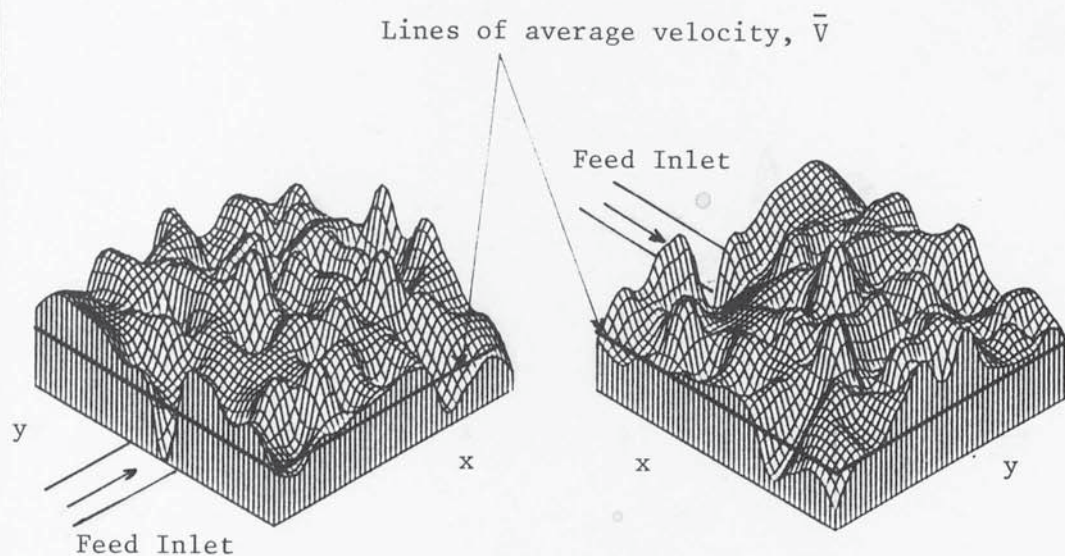


Fig. 7.2 Two views of Gas Distribution above an Ideally  
Distributed Bed of 381 mm diameter and 63.5 Packed  
Height

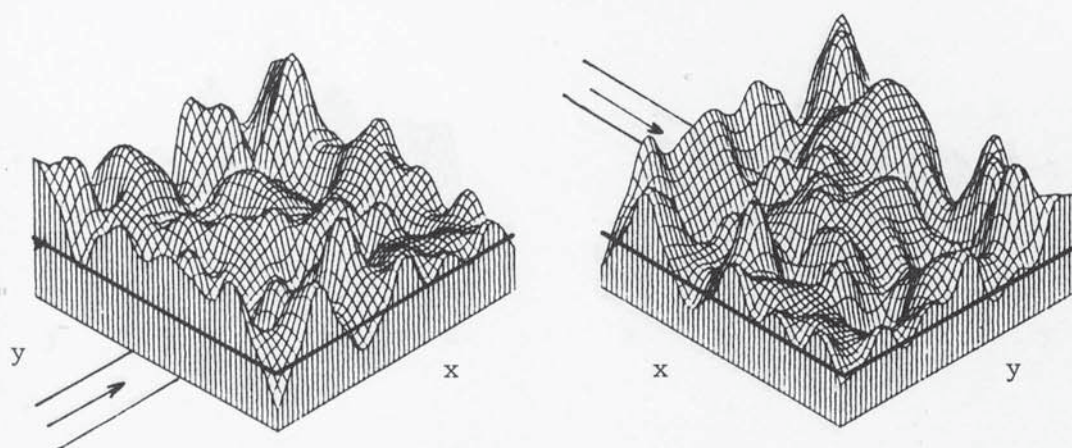


Fig. 7.3 Two Views of Gas Distribution above an Ideally  
Distributed Bed of 610 mm Diameter and 101.6 mm  
Packed Height

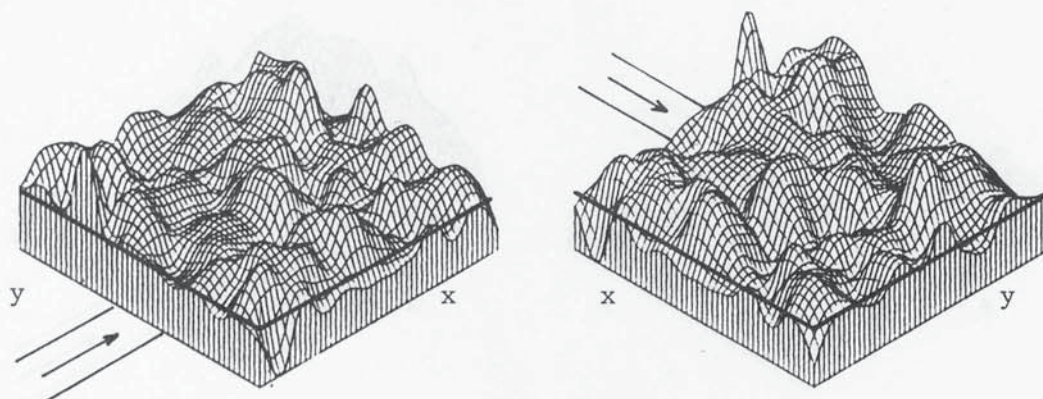


Fig. 7.4    Two views of Gas Distribution above an Ideally-  
Distributed Bed of 381 mm Diameter and 127 mm  
Packed Height

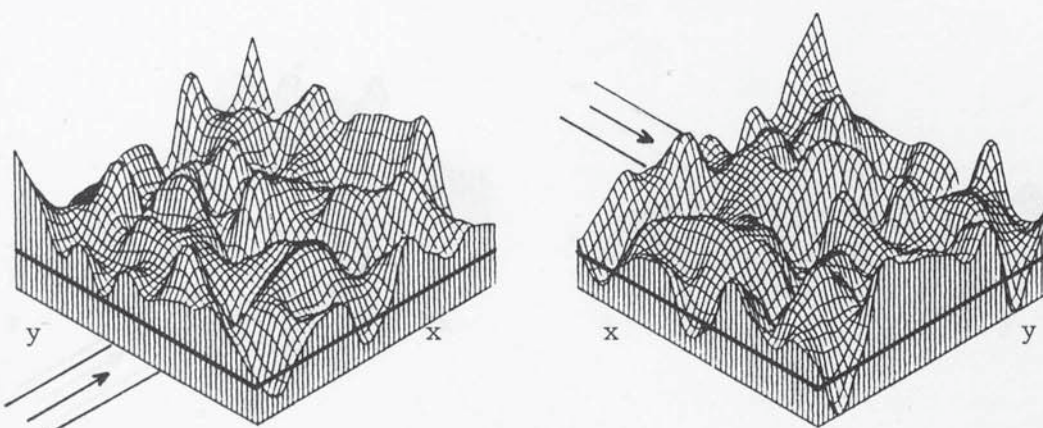


Fig. 7.5    Two views of Gas Distribution above an Ideally-  
Distributed Bed of 610 mm Diameter and 203 mm  
Packed Height



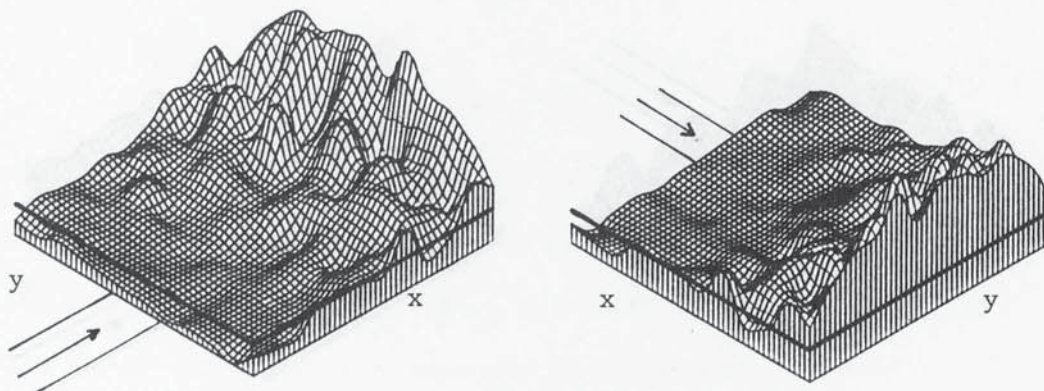


Fig. 7.6 Two views of Gas Distribution above a Maldistributed  
Bed of 381 mm Diameter and 63.5 mm Packed Height

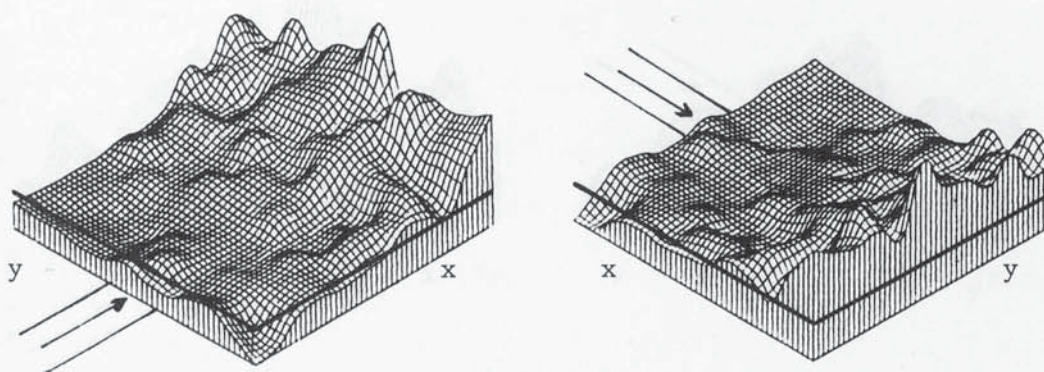


Fig. 7.7 Two Views of Gas Distribution above a Maldistributed  
Bed of 610 mm Diameter and 101.6 mm Packed Height

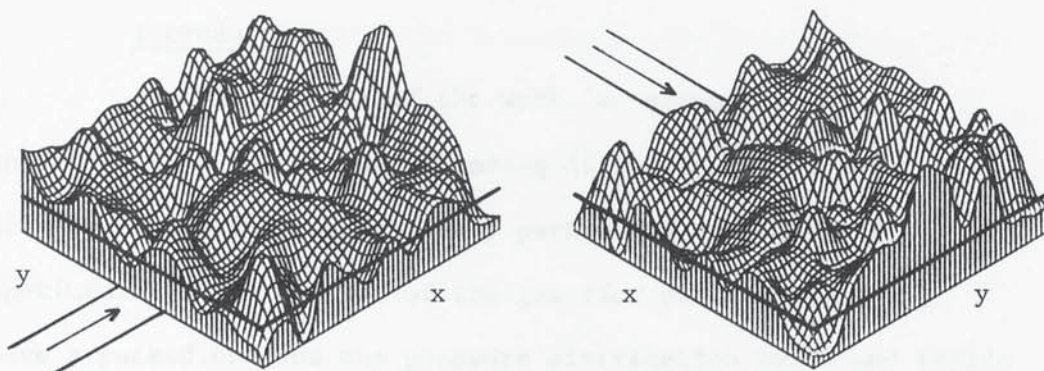


Fig. 7.8 Two views of Gas Distribution above a Maldistributed  
Bed of 381 mm Diameter and 127 mm Packed Height

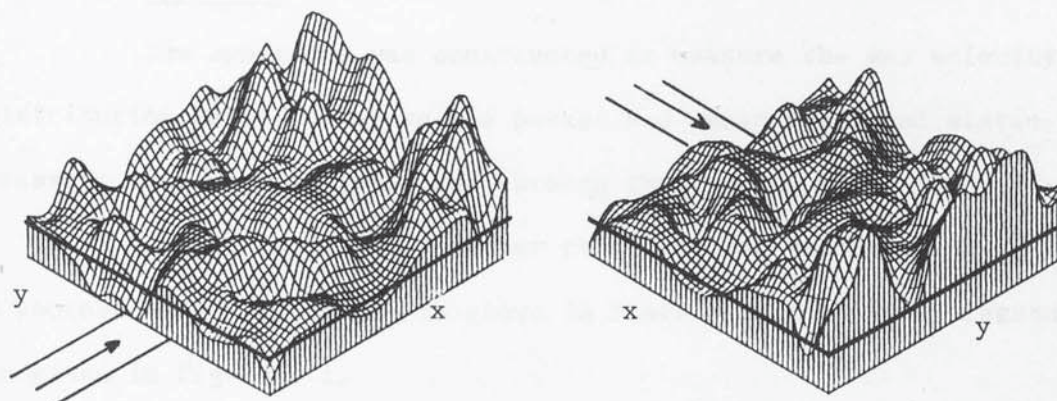


Fig. 7.9 Two Views of Gas Distribution above a Maldistributed  
Bed of 610 mm Diameter and 203 mm Packed Height



## SECTION VIII

### EXPERIMENTAL INVESTIGATION OF THE FLUID MECHANICS

#### OF MALDISTRIBUTED GAS FLOWS

##### Introduction

In this section of the work, an attempt is made to contribute to a better understanding of the causes of gas maldistribution in packed beds by performing an elaborate experimental investigation of the gas flow pattern below and above a packed bed and the pressure distribution below and inside that packed bed. The clarification of the mechanism of the distribution of the gas in packed beds will provide the basis for solving the problem of gas maldistribution in packed beds and the principles of designing devices to overcome this problem will be established.

##### 8.1 Apparatus

The apparatus was constructed to measure the gas velocity distribution below and above the packed bed under test, and static pressure distribution below and through that packed bed.

A 1220 mm inside diameter packed column was designed. A photograph of the column is shown in Plate 8.1 and a line diagram is given in Figure 8.1.

The shell is constructed from 3 mm thick mild steel sheets, it consists of two parts; a bottom part and a top part.

The bottom part, see Fig. 8.1, consists of an upside down truncated cone, 450 mm side length and 386 mm high. The small base, 756 mm inside diameter, is welded to a cylindrical part of 305 mm long whose other end is closed by a removable PVC plate. The big base of the cone is 1220 mm inside diameter welded to an 1320 outside

diameter flange. This part (bottom part) is held safely by a handy angle bar bench to a height of 600 mm above the ground to allow access for entering the column from beneath.

The top part, a 2000 mm long cylinder and 1220 mm inside diameter, is fitted with flanges from both ends and bolted to the bottom part.

The feed is introduced either tangentially or radially through a 152 mm inside diameter feed pipe whose central line is 400 mm above the bottom end of the cylindrical part.

A flat support plate of 1 cm x 1 cm square mesh made from 1 mm diameter wire is supported by a 1210 outside diameter support ring, 12.7 mm wide and 6.5 mm thick and fitted at 460 mm above the feed pipe central line to the inner side of the column. To keep the support plate flat when the bed is packed, a number of vertical 0.5 mm diameter galvanized steel wires are used to support the weight of the bed by hanging the support plate to a steel beam above the column. These galvanized wires exert a negligible, (if any), obstruction to the gas flowing inside the bed.

The static pressure below and inside the bed is measured by a micromanometer, described in Section 4.2.4, using an ordinary pitot-tube. The gas velocity below the bed is measured by the same micromanometer mentioned above by using a combination of an ordinary pitot-tube and 0.5 mm inside diameter steel tube described in Section 4.2.4 and shown in Plate 4.4. The gas velocity above the bed is measured by a modified hot-wire anemometer, described in Section 4.2.3.

The bed itself is packed with size 16 mm plastic pall rings to 6 cm and 36 cm depths.

Air is fed tangentially or radially by the air supply



equipment described in Section 4.1 at different flow rates.

The work in this section is performed in two stages according to the way of introducing the feed to the column.

## 8.2 Shallow Large Diameter Packed Beds with Tangential Feed Inlets

The air is introduced tangentially to the column at rates of  $0.814 \text{ m}^3/\text{sec.}$ ,  $0.706 \text{ m}^3/\text{sec.}$ , and  $0.59 \text{ m}^3/\text{sec.}$  below a bed packed to depths of 6 cm and 36 cm. The air velocity and the static pressure below, inside, and above the bed are measured along one diameter. It has been observed, below the bed, that the air is swirling tangentially and parallel to the bottom of the bed.

### 8.2.1 Results (Packed Depth = 6 cm)

The air was pumped at a rate of  $0.814 \text{ m}^3/\text{sec.}$ , i.e.  $44.5 \text{ m/sec}$  through the inlet pipe. It was observed that the feed temperature at the inlet of  $20^\circ\text{C}$  was raised to  $25^\circ\text{C}$  below and inside the bed. The static pressure below the bed, at the bottom of the bed and above the bed was measured and the air velocity below and above the bed was recorded along one diameter. These observations are listed in Table 8.1. The air velocity distribution is shown in Graph 8.1, the static pressure distribution is shown in Graph 8.2. The theoretical total number of heads of air just below the bed were compared with the actual heads measured, "static pressure", at the bottom of the bed in Graph 8.3.

The results show:

- 1) Below the bed, the air velocity is very high near the wall, especially the region opposite to the feed inlet, and

decreases towards the centre, see Graph 8.1.

2) The air, leaves the bed vertically, has a relatively high velocity near the wall and there is a negative flow in the middle part of the bed.

3) The static pressure below the bed is constant across the diameter and has a negative value of  $-0.025$  mm W.G., see Graph 8.2.

4) The static pressure just inside the bottom of the bed has high values near the wall and negative values at the area of the negative air flows.

5) The total theoretical number of heads (the pressure measured by the pitot-tube), below the bed is very high near the wall and falls sharply towards the centre and has values greater than the measured static pressure at the bottom of the bed, especially in the area near the feed inlet.

6) The static pressure inside the bed near the top has the same trend as that at the bottom but with lower values, a smaller variation and a larger area of negative pressure.

#### 8.2.2 Discussion of the Results (Packed Depth = 6 cm)

It is clear that the swirling air has a high velocity near the column wall which decreases towards the centre due to the nature of introducing the air. When this swirling gas, with such a velocity distribution, hits the packings at the bottom of the bed, its direction and its velocity are changed and some of its dynamic energy, (momentum), is transferred into a static pressure whose magnitude at a point depends on the velocity of the approaching swirling gas at that point. For that reason high

static pressure values are observed at the areas of high air velocities, i.e. near the wall. Theoretically, all the velocity heads just below the bed should be converted into a static pressure at the bottom of the bed, but, as shown in Graph 8.3, the measured static pressure is much less than the velocity heads. The difference between the theoretical and actual pressures is converted into a thermal energy due to the friction between the swirling air, on the one hand, and the support plate and the bottom layer of the packings, on the other hand. This explains the temperature rise from 20°C at the feed nozzle to 25°C inside the column.

Obviously, there should be a temperature variation in the bed, but this variation is unable to be detected due to the high rate of the gas mixing throughout and below the bed.

At the middle part of the bed where the swirling gas velocity is very low, a convergence between the values of the theoretical and actual pressures can be noticed, see Figure 8.3.

### 8.2.3 Negative Gas Flow in the Middle Area of the Packed Bed

The results show that the high static pressure is concentrated at the peripheral area of the bed, the width of this area is more or less equal to the diameter of the feed nozzle. This high pressure acts as a driving force which drives the air upwards, in that peripheral area, out of the bed at a quantity more than the feed inlet supplies. This results in a negative static pressure below the bed and negative air flows at the middle of the bed to maintain the mass balance.

#### 8.2.4 Results (Packed Depth = 36 cm)

The bed depth was increased to 36 cm and air velocity and pressure measurements were made at  $0.814 \text{ m}^3/\text{sec}$ ,  $0.71 \text{ m}^3/\text{sec}$  and  $0.6 \text{ m}^3/\text{sec}$  air volumetric flow rates. At air flow rate of  $0.814 \text{ m}^3/\text{sec}$ , the air velocity distribution below and above the bed is given in Graph 8.5. The static pressure distributions below the bed, at the bottom of the bed and at two levels inside the bed are shown in Graph 8.4 and listed in Table 8.2. The difference between the theoretical total number of the air heads and the measured static pressure is shown in Graph 8.6.

The static pressure is measured at the bottom of the bed and 15 cm above the bottom at air flow rates of  $0.71 \text{ m}^3/\text{sec}$  and  $0.6 \text{ m}^3/\text{sec}$  and listed in Tables 8.3 and 8.4 and plotted against the radial distance in Graphs 8.7 and 8.8 respectively.

The results show that:

- 1) The air below the bed has a high tangential velocity near the wall and decreases towards the centre, (see Fig. 8.5).
- 2) The air emerges from the top of the bed vertically and uniformly distributed except the point to point velocity variations, and there is no negative air flow.
- 3) The static pressure below the bed is constant and positive (+0.84 mm W.G.), see Fig. 8.4.
- 4) The static pressure distribution flattens as it moves up in the bed, and it is completely flat inside the bed near the top.
- 5) The theoretical number of heads of the air is higher than the measured static pressure near the wall; and lower than the static pressure at the middle area of the bed, (see Fig. 8.6).
- 6) The static pressure at a point in the bed increases



as the air velocity increases, (see Figs. 8.7 and 8.8).

### 8.3 Discussion

In a similar way to the experiments at a packed depth of 6 cm, the static pressure measured in the packing at the bottom of the bed is higher near the wall than that measured at the middle region due to higher air velocities near the wall. But neither the negative pressure nor the negative air flow is found in the middle region and the static pressure below the bed is positive and constant, (see Graph 8.4).

It is obvious that the pressure difference between two areas is the force which drives the air from the high pressure area to the low pressure area in the packed bed. This means that in addition to the upwards, "axial", flow of the air in the wall zone, (High Pressure Zone), some air is flowing "Radially" towards the middle area, (Low Pressure Zone).

In the 6 cm deep bed there was not enough packing depth to homogenize the pressure and the air flow. But the 36 cm depth has allowed for the homogenizing of the pressure, and consequently the air flows, to take place before emerging from the top of the bed. For this reason the pressure profile becomes flatter at higher levels in the 36 cm deep bed, until it becomes flat at a distance of 30 cm above the bottom of the bed. Above this level, the pressure profile has maintained its flatness causing the air to emerge uniformly from the top of the bed, (see Graph 8.5).

It is clear that the theoretical number of heads of the air below the bed is higher than the actual measured static pressure inside the packing near the bottom due to the friction energy losses.

This phenomenon is clearly observed in the 6 cm deep packed bed, but in the 36 cm deep bed, the situation is the same in the wall zone but it is apparently reversed in the middle area of the bed, (see Graph 8.6). In reality the actual static pressure in the middle area is also less than the theoretical pressure, but the measured static pressure shown in Graph 8.6 is the summation of the static pressure below the bed due to the bed resistance to air flow, and the velocity generated static pressure. Thus the summation of these two types of the static pressure is larger than the theoretical number of heads of the air at the middle area which is small due to the low tangential air velocity below this area.

#### 8.4 Comments on the Velocity Generated Static Pressure

##### Distribution

As shown above, the velocity generated static pressure at a point inside the packing near the bottom of the bed equals the measured static pressure at that point minus the static pressure below the bed, which has a constant value. This means that the velocity generated static pressure has a similar distribution to that of the measured static pressure. Thus, the following comments can be made:

- 1) The velocity generated static pressure at a point inside the packing near the bottom of the bed is proportional to the velocity of the swirling air below the bed approaching that point. That is proportional to the velocity of air in the feed pipe. This is shown in Graph 8.9, where the value of velocity generated static pressure at different points at the bottom of the bed is drawn against the air velocity (square) in the feed pipe. The straight lines indicate that:



$$P_g \propto V_p^2 \quad \text{..... (8.1)}$$

where  $P_g$  is the velocity generated static pressure.

$V_p$  is the air velocity in the feed pipe.

Eqn. 8.1 assumes a constant air density,  $\rho$ . Thus to generalize the equation:

$$P_g = k\rho(V_p)^2 \quad \text{..... (8.2)}$$

Where  $k$  is the proportion of the theoretical total number of heads transformed into a static pressure at the bottom of the bed.

2) The velocity generated static pressure at the bottom of the packed bed is more or less the same in the small depth bed, (6 cm), and the large depth, (36 cm). This is shown in Graph 8.10 where the value of  $P_g$  for the two beds is drawn against the radial distance; [the values of  $P_g$  are obtained by subtracting the static pressure below each bed from the measured static pressure at the bottom of that bed respectively. These values are listed in Tables 8.1 and 8.2].

Therefore  $k$  can be assumed to be the same for both bed depths.

3) The width of the high pressure area in both beds, (the 6 cm and 36 cm deep beds), is constant and more or less equals the diameter of the feed pipe, (see Graph 8.10), where the width of high pressure zone equals the feed inlet diameter, (15.2 cm).

## 8.5 Pressure Distribution in Shallow Large Diameter Packed Beds with Radial Feed Inlets

The air is fed radially through 15.2 cm diameter pipe

below the 36 cm deep packed bed at a rate of  $0.6 \text{ m}^3/\text{sec}$ . The static pressure is measured, along one diameter in line with the feed direction, at the bottom of the bed and at 15 cm above the bottom. The results are listed in Table 8.5 and drawn against the radial distance in Graph 8.11.

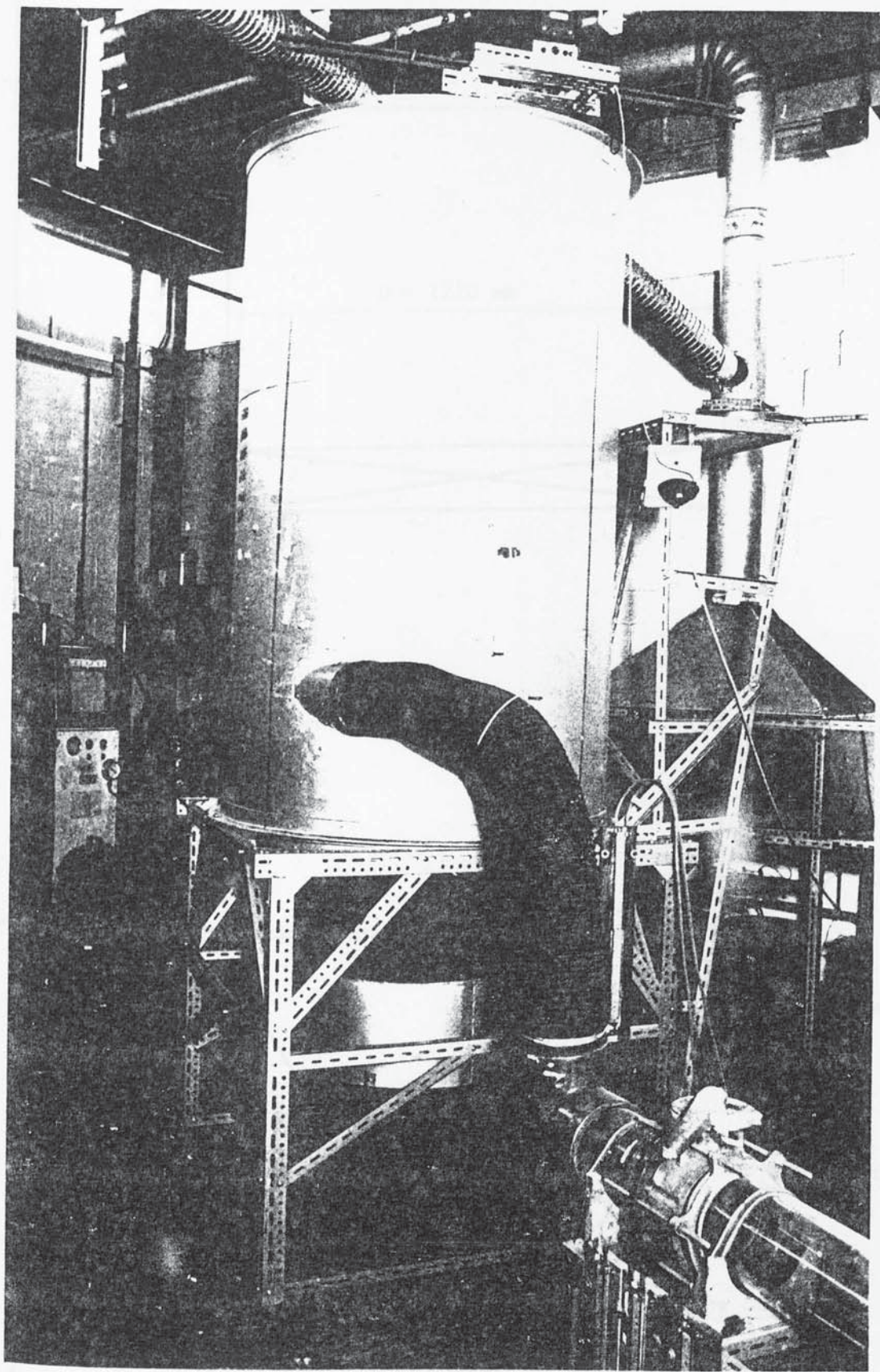
The results show a very high static pressure in the area opposite to the feed inlet and a relatively high static pressure at the area near the feed inlet. The smallest static pressure was observed in the middle area. The static pressure profile flattens at higher levels in the bed.

#### 8.6 Summary

The air velocity and static pressure distributions are observed in packed beds with a tangential feed inlet, in this section of the work. The low pressure and the high pressure zones were observed. A negative air flow was noticed at very short packed depths which disappears as the packed depth increases.

In the next stage of the work, simple models have been suggested to explain the flow patterns in packed beds based on the results observed in this section.

PLATE 8.1





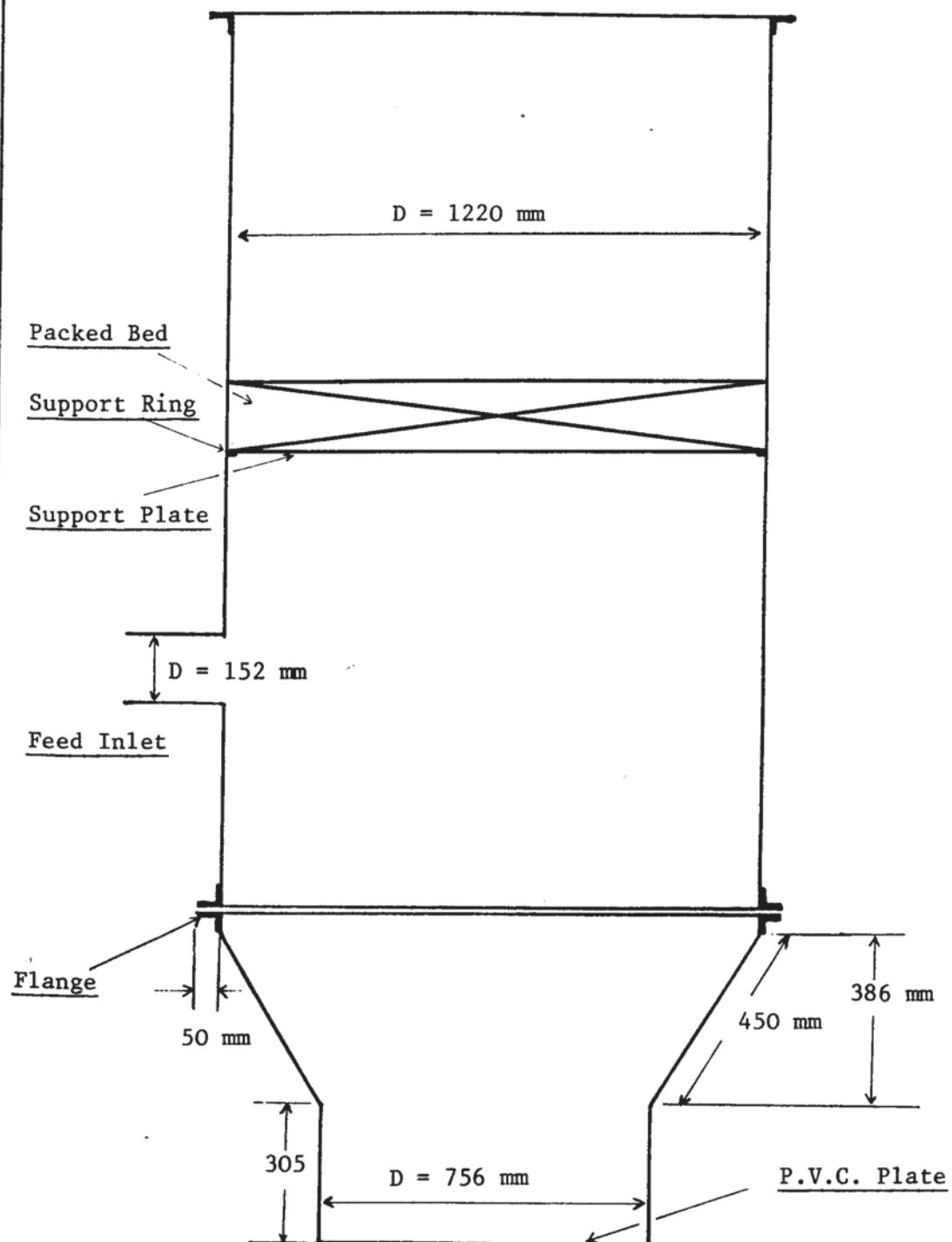


Fig. 8.1 Diagram of the Apparatus used in measuring Gas velocity and Pressure Distributions



Table 8.1 Pressure Distribution in a 6cm Deep Packed Bed

(Air flow rate =  $0.814 \text{ m}^3/\text{sec}$ )

r(cm)	Static Pressure (mm W.G.)			Below the Bed	
	Below the Bed	Distance above the Bottom (cm)		Pitot-tube Reading	Velocity m/sec.
		0	6		
60	-0.025	2.6	0.6	14.85	15.572
54	-0.025	2.3	0.5	14.25	15.254
48	-0.025	1.0	0.30	10.50	13.090
42	-0.025	0.30	0.10	5.90	5.815
36	-0.025	0.00	-0.15	2.00	5.715
30	-0.025	-0.06	-0.2	0.81	3.637
24	-0.025	-0.08	-0.38	0.60	3.130
18	-0.025	-0.09	-0.42	0.24	1.58
12	-0.025	-0.11	-0.42	0.09	1.202
6	-0.025	-0.14	-0.42	0.03	0.70
C	-0.025	-0.14	-0.42	0.02	0.571
6	-0.025	-0.14	-0.42	0.06	0.99
12	-0.025	-0.13	-0.40	0.3	2.213
18	-0.025	-0.12	-0.32	0.45	2.711
24	-0.025	0.075	-0.24	1.2	4.276
30	-0.025	0.10	0.0	5.4	9.39
36	-0.025	0.75	0.1	7.2	10.843
42	-0.025	1.1	0.2	12.0	14.0
48	-0.025	3.6	1.0	13.5	14.847
54	-0.025	4.0	2.0	14.1	15.173
60	-0.025	8.0	4.8	15.9	16.113

Table 8.2 Pressure Distribution in a 36 cm Deep Bed

(Air flow rate =  $0.814 \text{ m}^3/\text{sec}$ )

r (cm)	Static Pressure (mm W.G.)				Below the Bed	
	Below the Bed	Distance above the Bottom (cm)			Pitot-tube Reading	Velocity m/sec
		0	15	30		
60	0.84	5.20	1.90	0.35	15.0	15.65
54	0.84	4.10	1.80	0.30	14.4	15.334
48	0.84	2.45	1.30	0.30	10.8	13.28
42	0.84	1.70	1.20	0.25	5.4	9.4
36	0.84	1.30	0.99	0.25	2.2	6.0
30	0.84	0.96	0.69	0.20	0.8	3.614
24	0.84	0.90	0.69	0.19	0.45	2.711
18	0.84	0.81	0.69	0.18	0.12	1.4
12	0.84	0.84	0.66	0.18	0.08	1.143
6	0.84	0.80	0.64	0.18	0.03	0.7
C	0.84	0.77	0.63	0.18	0.02	0.571
6	0.84	0.80	0.60	0.18	0.20	1.807
12	0.84	0.84	0.65	0.18	3.6	7.667
18	0.84	0.90	0.60	0.19	2.1	5.856
24	0.84	1.00	0.65	0.20	3.4	7.451
30	0.84	1.50	0.90	0.25	6.9	10.615
36	0.84	2.75	1.35	0.25	9.6	12.52
42	0.84	4.30	2.0	0.28	12.6	14.343
48	0.84	6.50	2.5	0.35	13.8	15.01
54	0.84	7.70	2.75	0.50	14.1	15.173
60	0.84	8.50	2.85	0.60	15.3	15.80

Table 8.3 Pressure Distribution in 36 cm Deep Bed

(Air flow rate =  $0.706 \text{ m}^3/\text{sec}$ )

r(cm)	Static Pressure (mm W.G.)			Below the Bed	
	Below the Bed	Distance above the Bottom (cm)		Pitot-tube Reading	Velocity m/sec
		0	15		
60	0.53	4.1	1.5	11.2	13.52
54	0.53	3.22	1.43	10.7	13.2
48	0.53	2.15	1.07	8.1	11.9
42	0.53	1.35	0.95	4.35	8.43
36	0.53	0.98	0.79	1.7	5.27
30	0.53	0.7	0.56	0.7	3.38
24	0.53	0.63	0.5	0.36	2.42
18	0.53	0.58	0.5	0.12	1.4
12	0.53	0.55	0.49	0.08	1.14
6	0.53	0.54	0.47	0.035	0.75
C	0.53	0.54	0.47	0.025	0.64
6	0.53	0.57	0.45	0.2	1.8
12	0.53	0.61	0.49	0.9	3.83
18	0.53	0.67	0.51	1.9	5.57
24	0.53	0.77	0.55	3.2	7.23
30	0.53	1.23	0.74	5.95	9.85
36	0.53	2.25	1.08	8.1	11.5
42	0.53	3.5	1.57	10.0	12.78
48	0.53	5.1	2.0	16.85	13.31
54	0.53	6.1	2.3	11.45	13.67
60	0.53	6.7	2.37	12.5	14.28

Table 8.4 Pressure Distribution in 36 cm Deep Bed

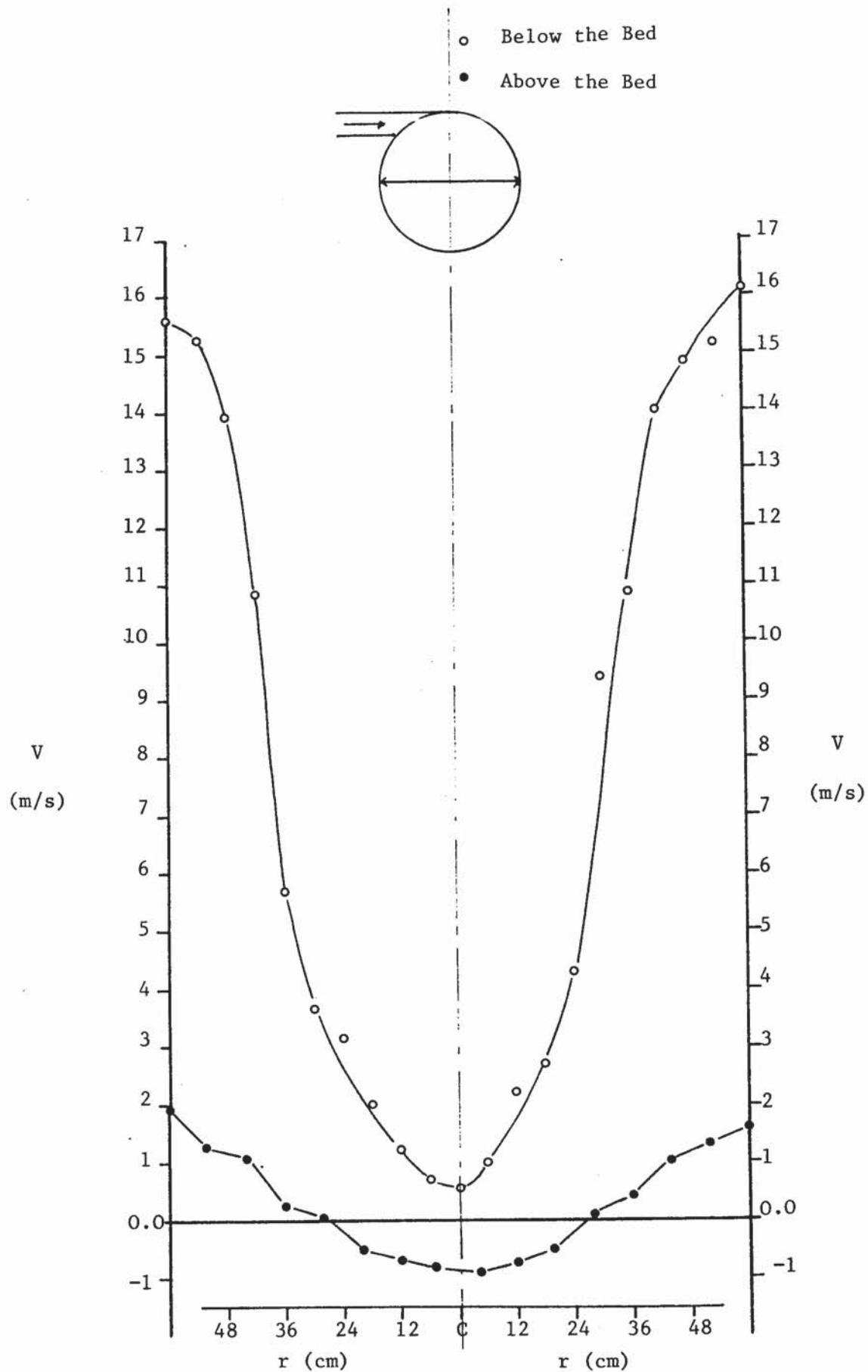
(Air flow rate =  $0.59 \text{ m}^3/\text{sec}$ )

r(cm)	Static Pressure (mm W.G.)			Below the Bed	
	Below the Bed	Above the Bottom (cm)		Pitot-tube Reading	Velocity
		0	15		
60	0.21	2.8	1.02	7.4	11
54	0.21	2.2	0.58	7.0	10.69
48	0.21	1.2	0.8	5.4	9.39
42	0.21	0.93	0.65	3.3	7.34
36	0.21	0.6	0.55	1.2	4.42
30	0.21	0.38	0.40	0.6	3.13
24	0.21	0.32	0.35	0.27	2.1
18	0.21	0.30	0.29	0.12	1.4
12	0.21	0.29	0.28	0.08	1.143
6	0.21	0.28	0.28	0.04	0.81
C	0.21	0.28	0.28	0.03	0.7
6	0.21	0.3	0.28	0.20	1.81
12	0.21	0.35	0.3	1.0	4.04
18	0.21	0.4	0.35	1.7	5.27
24	0.21	0.5	0.43	3.0	7.0
30	0.21	0.9	0.55	5.0	9.03
36	0.21	1.65	0.75	6.6	10.38
42	0.21	2.58	1.05	7.4	11
48	0.21	3.4	1.4	7.1	10.77
54	0.21	4.2	1.75	8.0	11.43
60	0.21	4.6	1.8	8.6	11.85

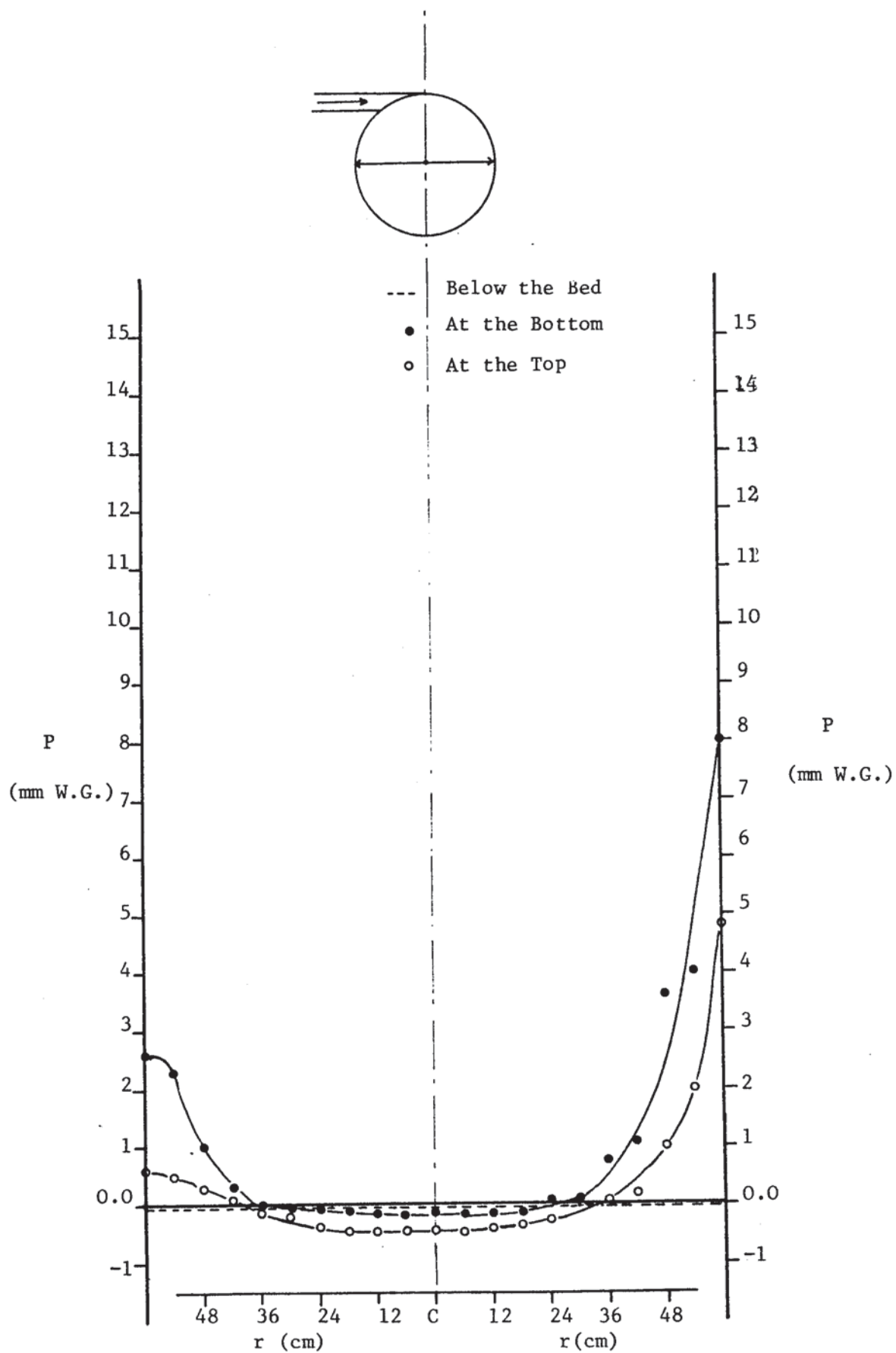
Table 8.5    Pressure Distribution in a 36 cm Deep Bed  
with a Radial Feed Inlet  
(Air flow rate = 0.59 m<sup>3</sup>/sec)

Distance from the feed side	Static Pressure (mm W.G.)		
	below the bed	Distance above the bed	
		0	15
0	0.21	1.9	0.85
12	0.21	1.6	0.85
24	0.21	1.4	0.75
36	0.21	1.35	0.80
48	0.21	1.20	0.70
60	0.21	1.15	0.70
72	0.21	1.0	0.70
84	0.21	1.2	0.80
96	0.21	1.7	1.0
108	0.21	3.0	1.3
120	0.21	5.6	1.6

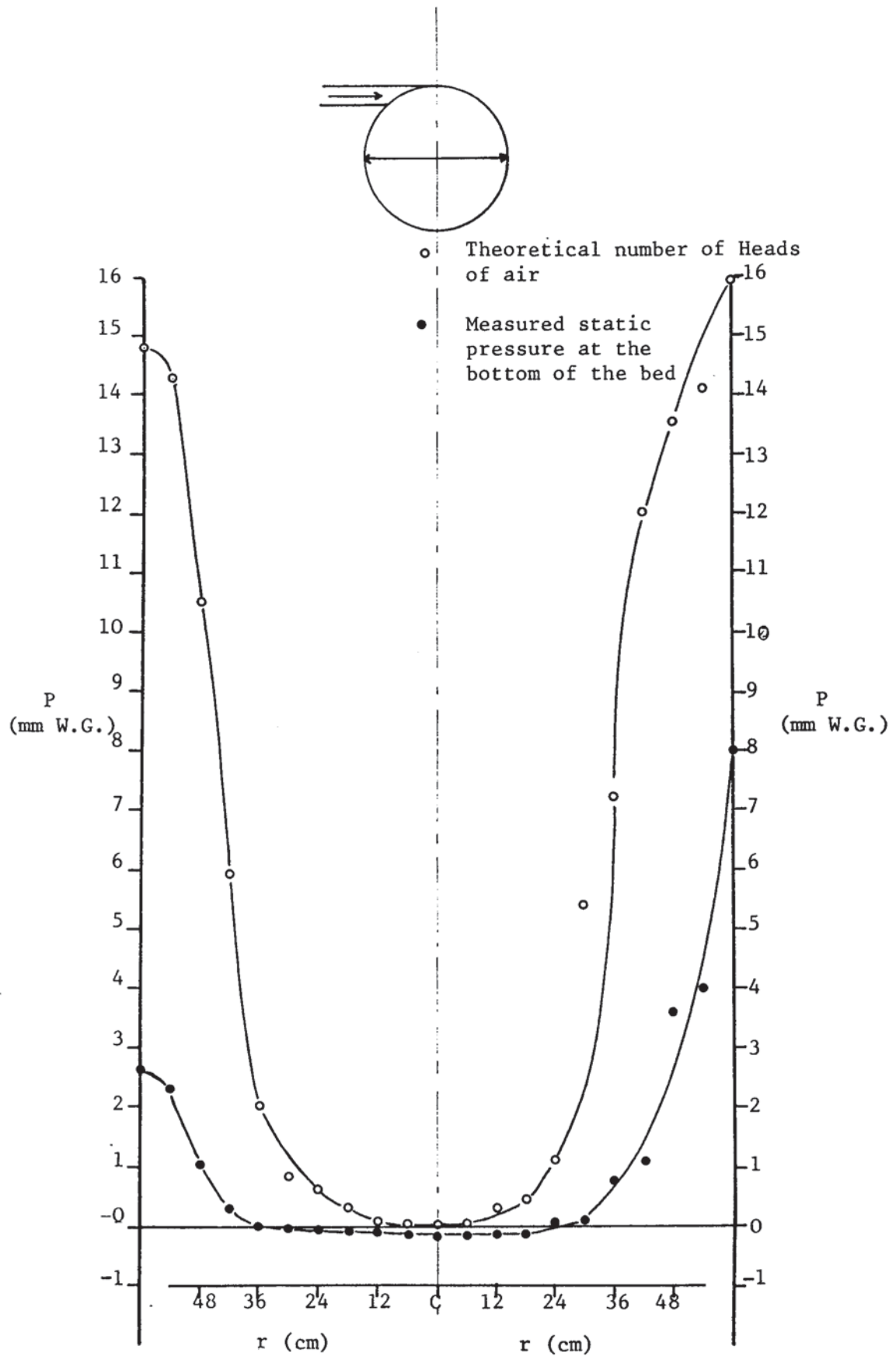




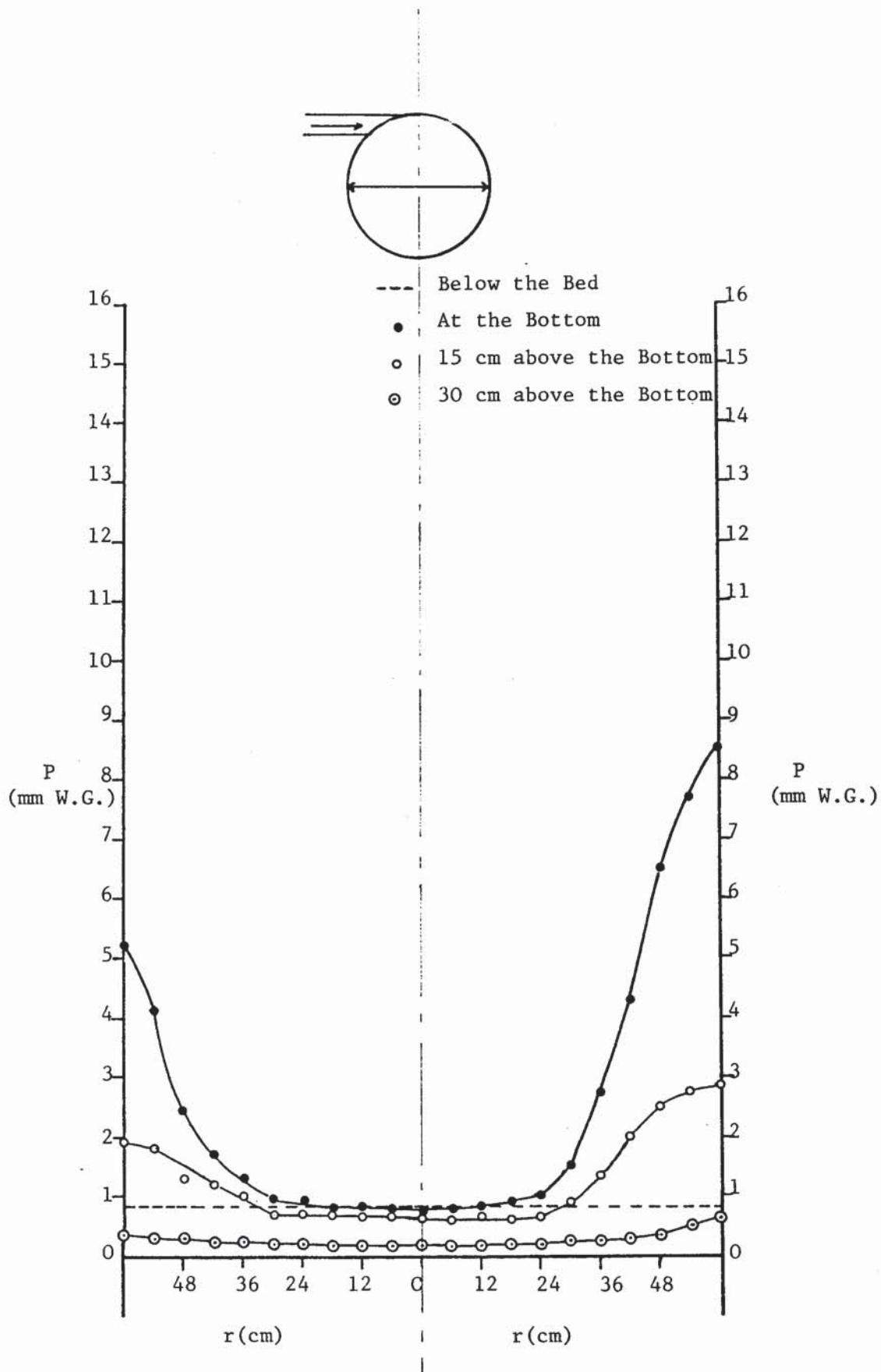
Graph 8.1    Velocity Distribution in a 6cm Deep Bed



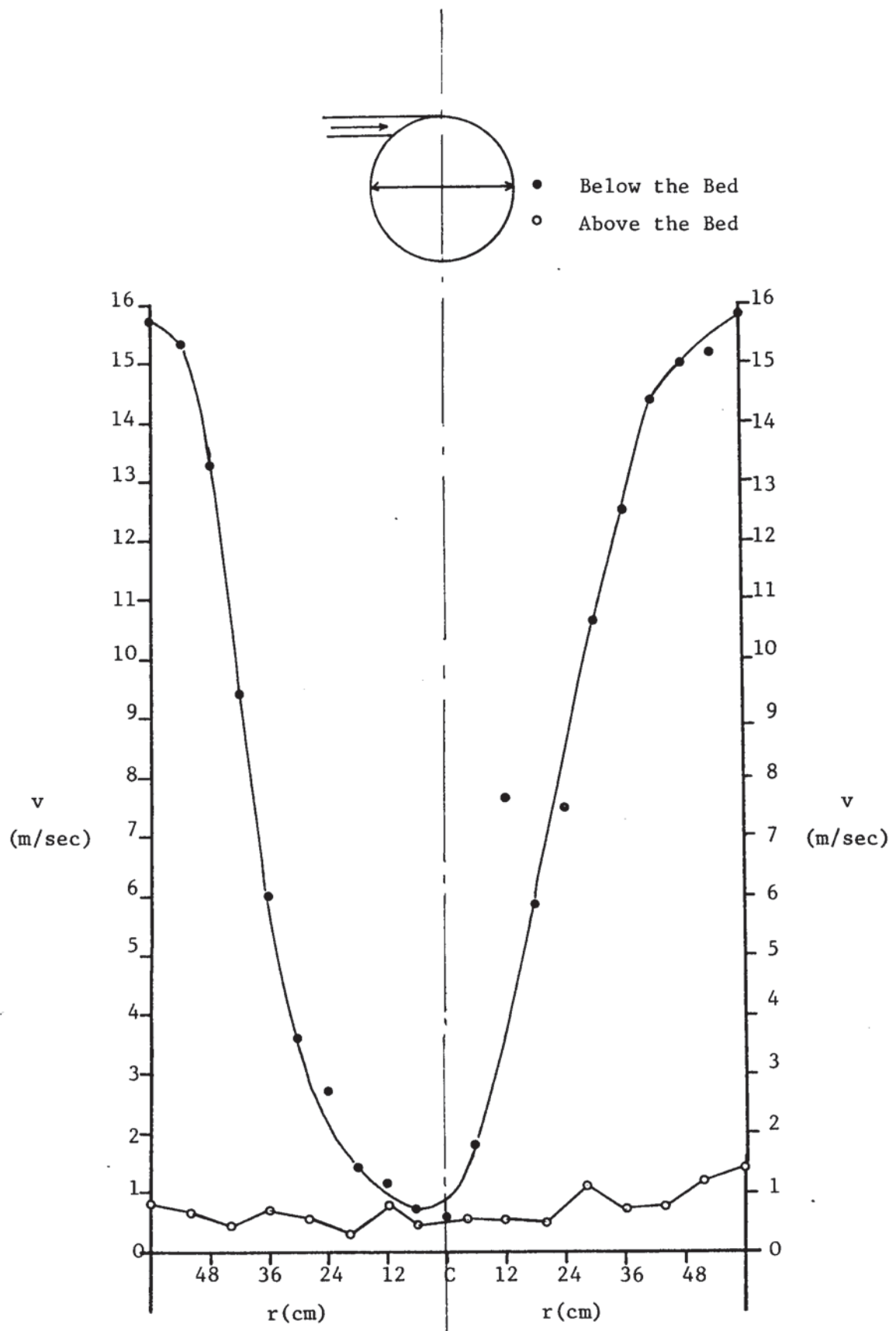
Graph 8.2 Pressure Distribution in a 6 cm Deep Bed



Graph 8.3    Theoretical and Measured Number of Heads  
in the 6cm Deep Bed

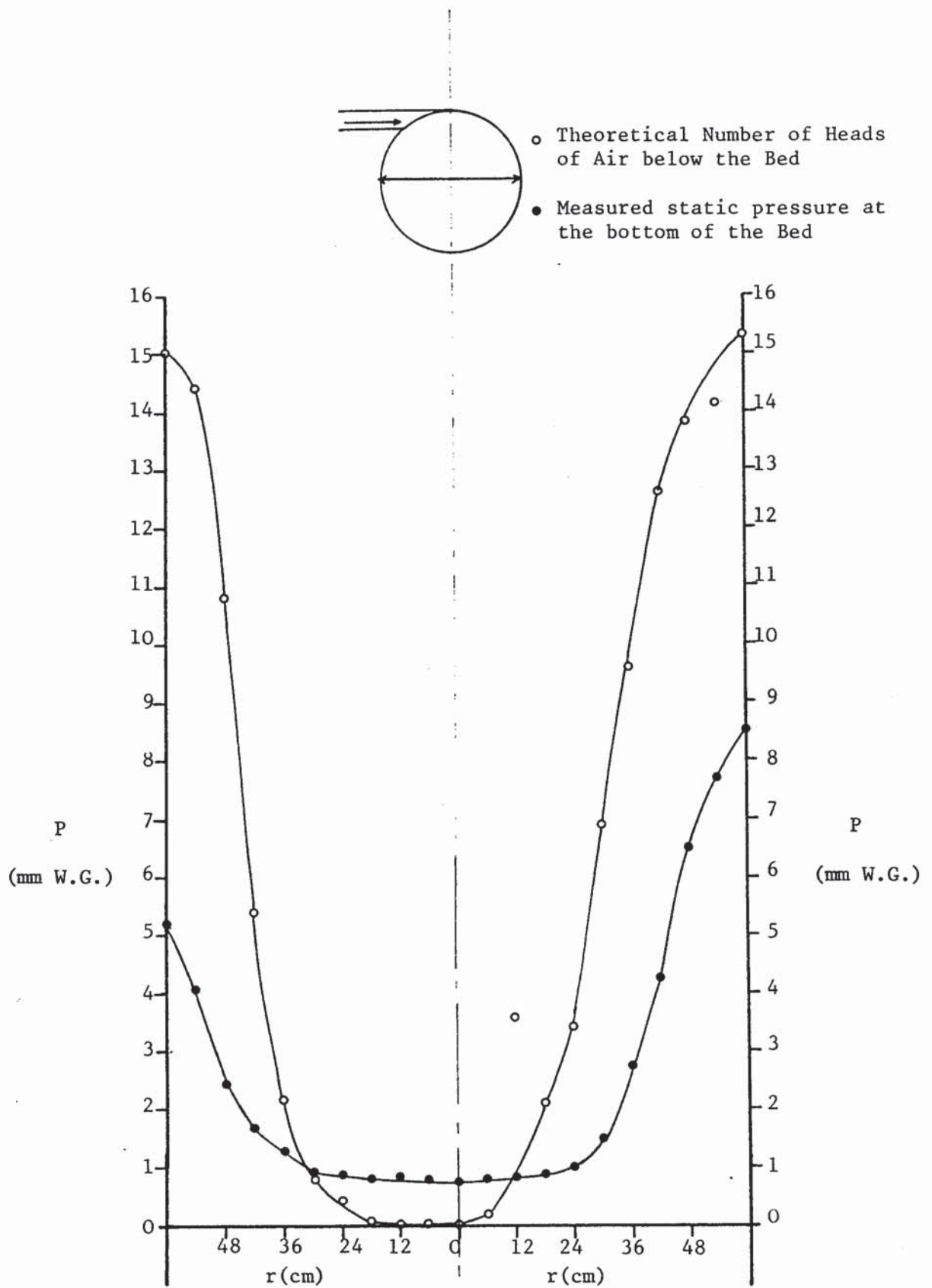


Graph 8.4    Pressure Distribution in a 36 cm Deep Bed

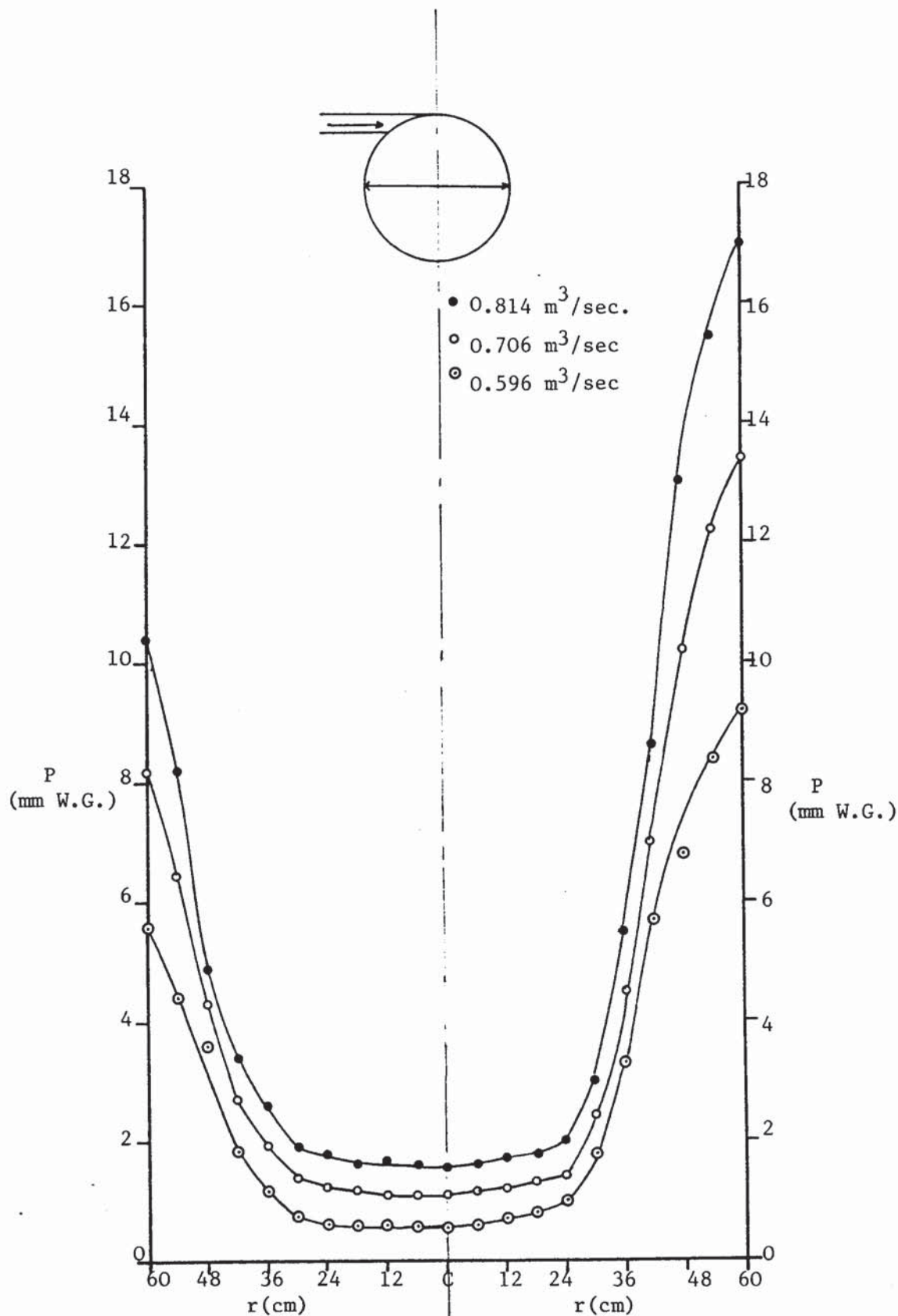


Graph 8.5    Air Velocity Distribution in the 36 cm  
Deep Bed

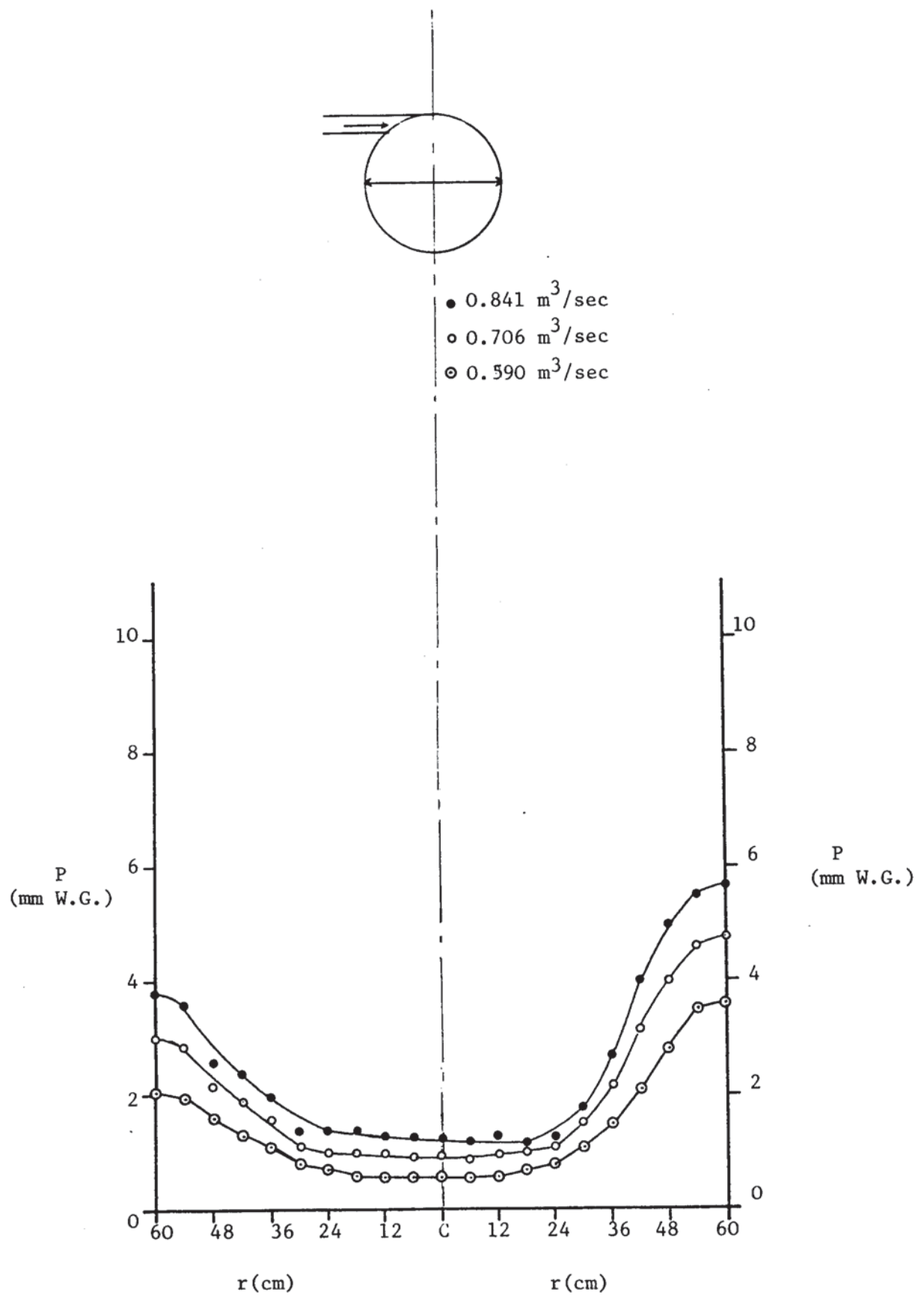




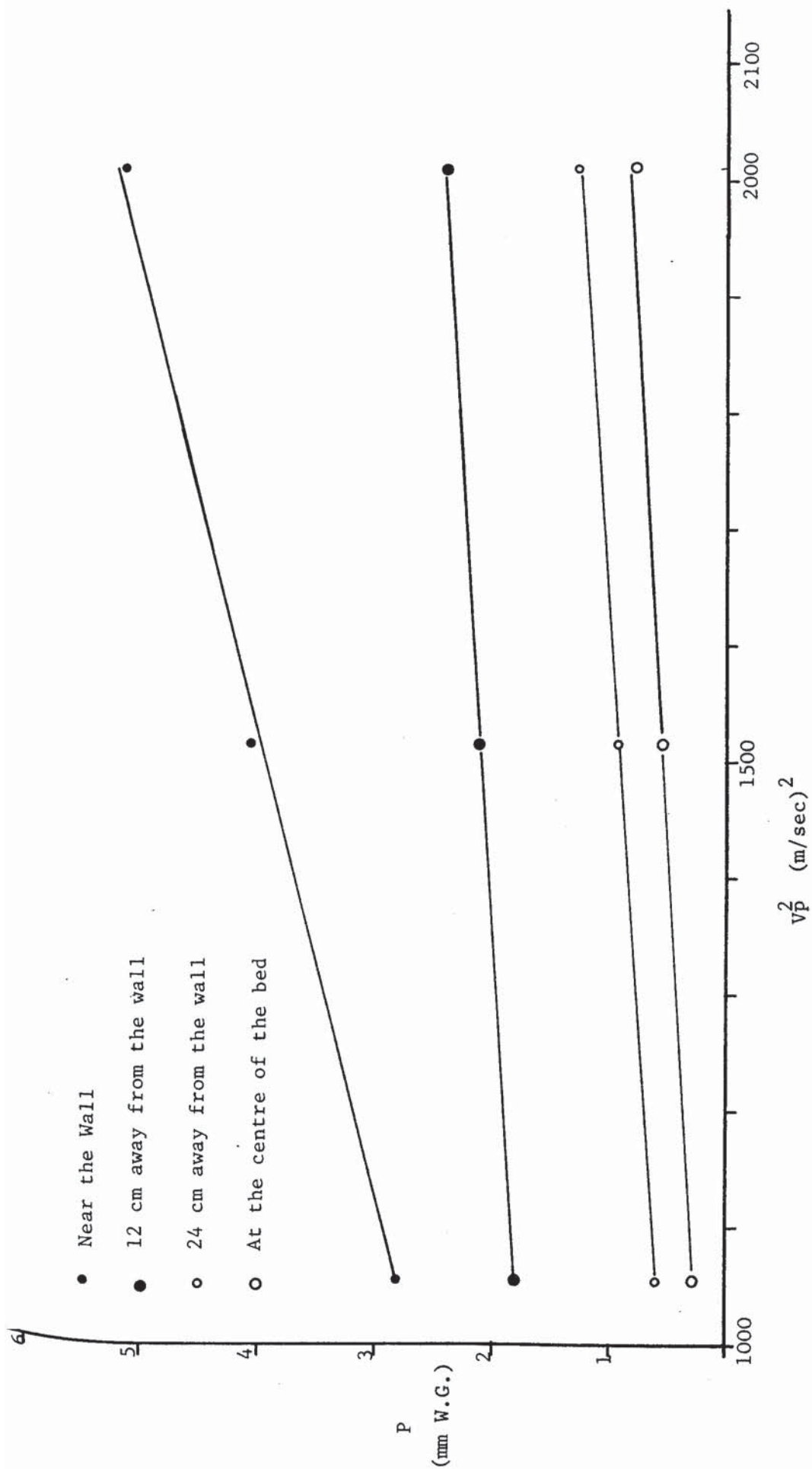
Graph 8.6    Theoretical and Measured Number of Heads  
in the 36 cm Deep Bed



Graph 8.7 Pressure Distribution at the Bottom of  
 36 cm Deep Bed at Different Air Flows



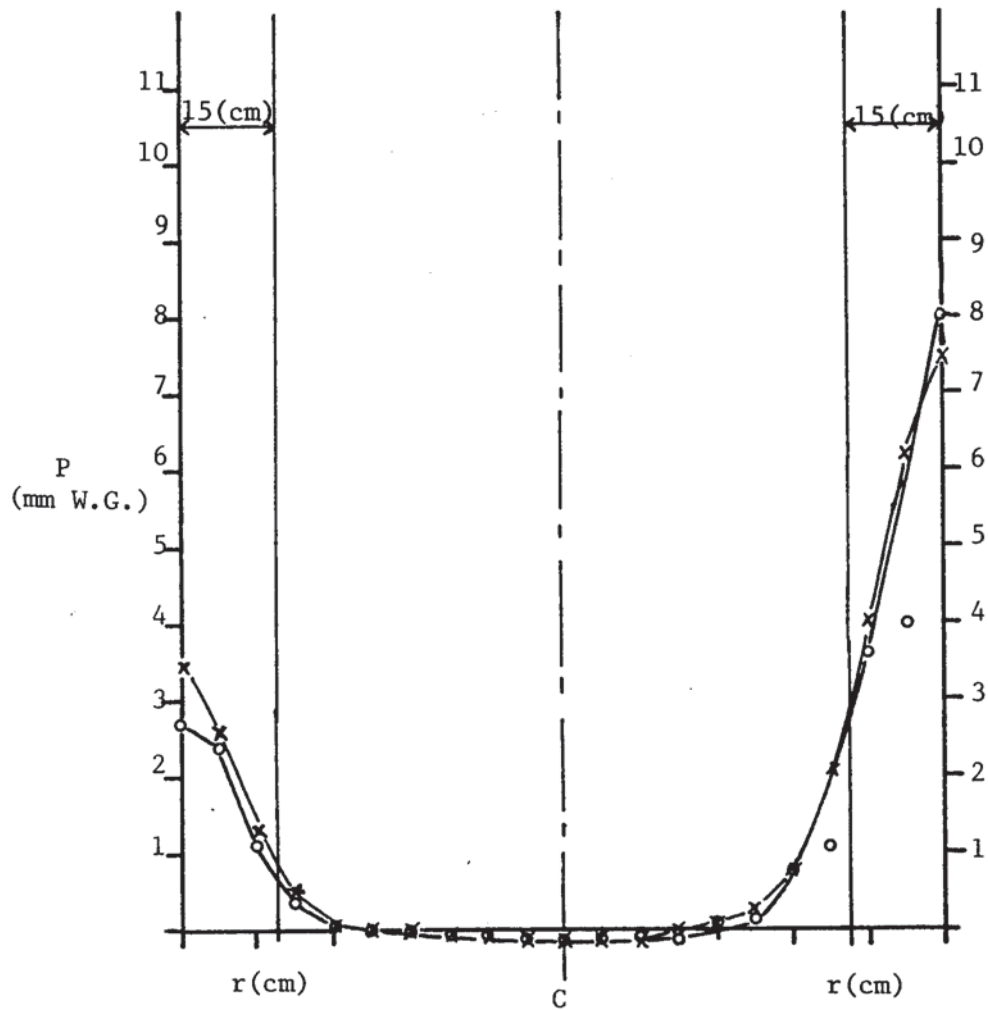
Graph 8.8    Pressure distribution at a Distance of 15 cm  
above the Bottom of 36 cm Deep Bed at Different  
Air Flows



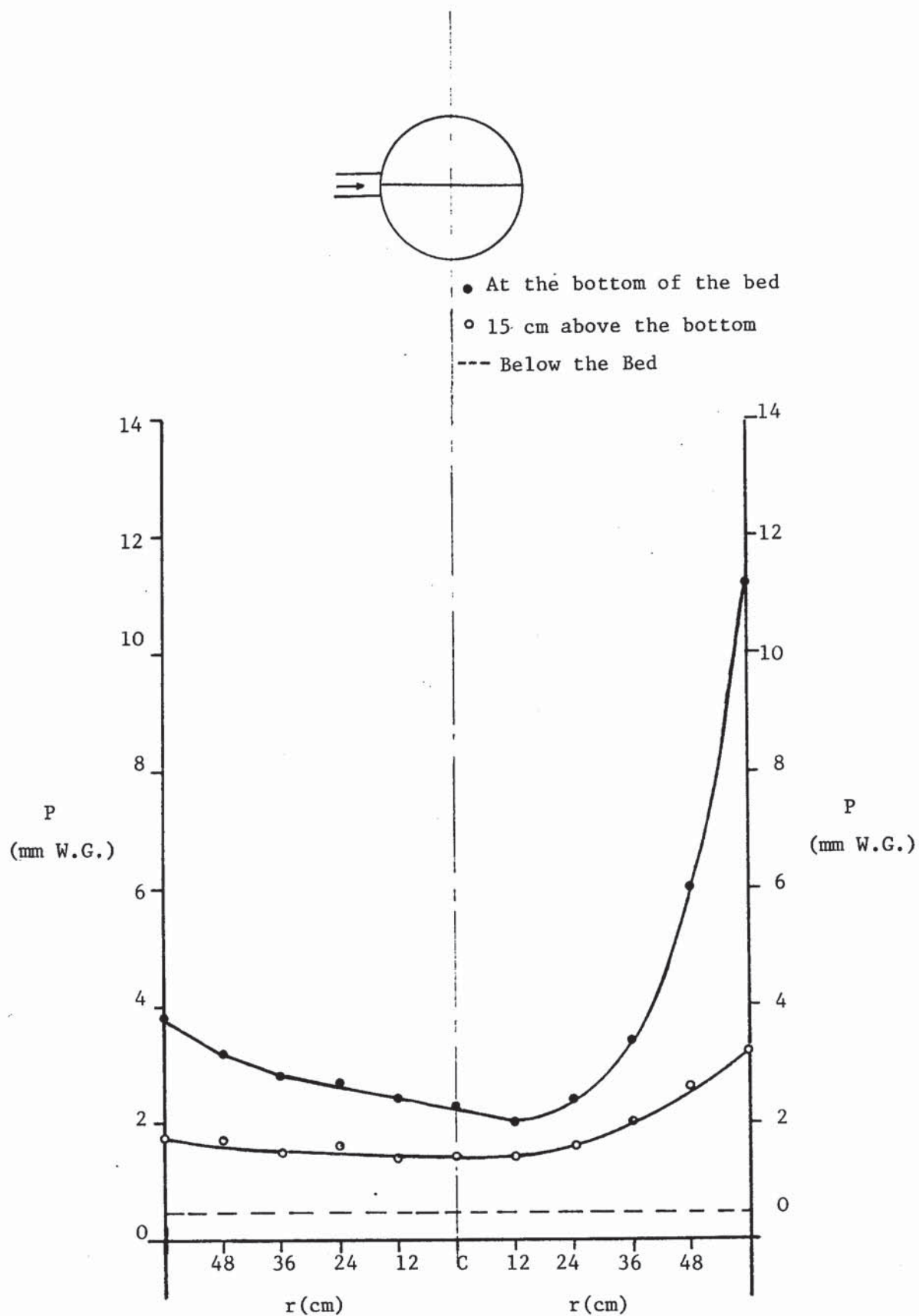
Graph 8.9 Variation of the static pressure generated at the bottom of 36 cm deep packed bed with air velocity in the feed inlet

x Packed Depth = 36 cm

o Packed Depth = 6 cm



Graph 8.10    The Velocity Generated Static Pressure at the Bottom  
Of Packed Beds with a Tangential Feed Inlet, 15.2 cm  
in Diameter



Graph 8.11    Pressure Distribution in a Radially Fed Bed  
Packed to a Depth of 36 (cm)



SECTION IX  
THEORETICAL MODELS OF MALDISTRIBUTED GAS FLOW  
THROUGH PACKED BEDS

Introduction

The work has shown up to now that the gas is maldistributed through packed beds. This maldistribution ranges from a very mild situation of point to point velocity variation due to the channelling effect, to a very severe situation where the upwards flow near the wall of the column is accompanied by downwards flow at the middle of the bed.

In this section of the work, an attempt has been made to formulate models to describe this maldistribution of the gas in packed beds so as to support the scale-up rules obtained in Section 5, and to identify packed column designs subject to gas maldistribution. Three models have been described; a model based on three dimensional flow of gas through packed beds, which is more exact but too difficult for analytical solution, and two simplified models which are based on flow in one or two directions which permit the derivation of equations which aid interpretation of the observed flow patterns and which confirm the rules of scale-up proposed in earlier sections.

9.1 Three-dimensional Gas Flow Through Packed Beds

Consider an elemental volume  $dx dy dz$  in the bed, where  $dx$ ,  $dy$  and  $dz$  are the width, depth and height of the element shown in Fig. 9.1. For an incompressible fluid flowing in three dimensions through this element the  $x$ ,  $y$ , and  $z$  components of the differential versions of the Ergun equation are written as:

$$-\frac{\partial P}{\partial x} = \frac{A\mu}{\rho} g_x + \frac{B}{\rho} g_x^2 \quad \dots\dots (9.1a)$$

$$-\frac{\partial P}{\partial y} = \frac{A\mu}{\rho} g_y + \frac{B}{\rho} g_y^2 \quad \dots\dots (9.1b)$$

$$-\frac{\partial P}{\partial z} = \frac{A\mu}{\rho} g_z + \frac{B}{\rho} g_z^2 \quad \dots\dots (9.1c)$$

where

$$A = \frac{150(1-\epsilon)^2}{\epsilon^3 \phi dp}$$

and

$$B = \frac{1.75 (1-\epsilon)}{\epsilon^3 \phi dp}$$

P = Pressure

$\mu, \rho$  = Gas viscosity and density

$g_x, g_y, g_z$  = Gas mass velocities in x, y and z directions

$\epsilon$  = Void fraction

$\phi$  = Shape factor

dp = Particle size

Gas flow through this elemental volume may be laminar, turbulent or laminar in one part and turbulent in the other part of the element. These three situations are to be studied separately in detail.

#### 9.1.1 Laminar flow

For laminar gas flow through the elemental volume  $dx dy dz$ , equations 9.1a, 9.1b and 9.1c are reduced to

$$-\frac{\partial P}{\partial x} = \frac{A\mu}{\rho} g_x \quad \dots\dots (9.2a)$$

$$-\frac{\partial P}{\partial y} = \frac{A\mu}{\rho} g_y \quad \dots\dots (9.2b)$$

$$-\frac{\partial P}{\partial z} = \frac{A\mu}{\rho} g_z \quad \dots\dots (9.2c)$$

Obtaining  $g_x$ ,  $g_y$  and  $g_z$  from these equations:

$$g_x = -a \frac{\partial P}{\partial x}$$

$$g_y = -a \frac{\partial P}{\partial y}$$

$$g_z = -a \frac{\partial P}{\partial z}$$

where

$$a = \frac{\rho}{A\mu}$$

Performing a mass balance over the elemental volume for no accumulation or depletion results in the following equation, (see Fig. 9.1),

$$\left[ -a \frac{\partial P}{\partial x} dydz + a \left( \frac{\partial P}{\partial x} + \frac{\partial^2 P}{\partial x^2} dx \right) dydz \right] + \left[ -a \frac{\partial P}{\partial y} dx dz + a \left( \frac{\partial P}{\partial y} + \frac{\partial^2 P}{\partial y^2} dy \right) dx dz \right]$$

$$+ \left[ -a \frac{\partial P}{\partial z} dx dy + a \left( \frac{\partial P}{\partial z} + \frac{\partial^2 P}{\partial z^2} dz \right) dx dy \right] = 0$$

$$\therefore a \left[ \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} \right] dx dy dz = 0$$

and finally

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = 0 \quad \dots\dots (9.3)$$

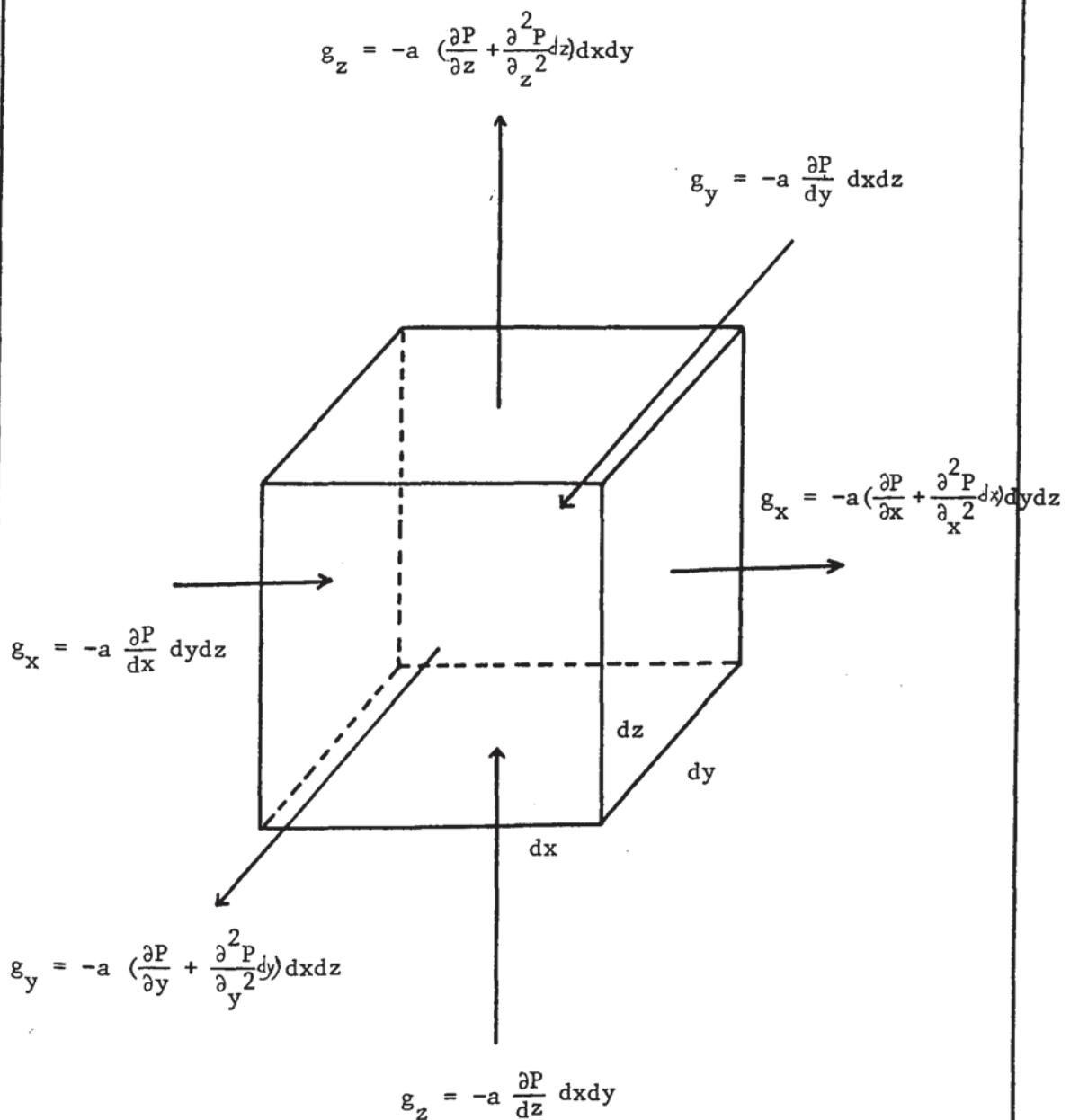


Figure 9.1. Mass Balance Over the (dx dy dz) Elemental Volume  
in Laminar Gas Flow

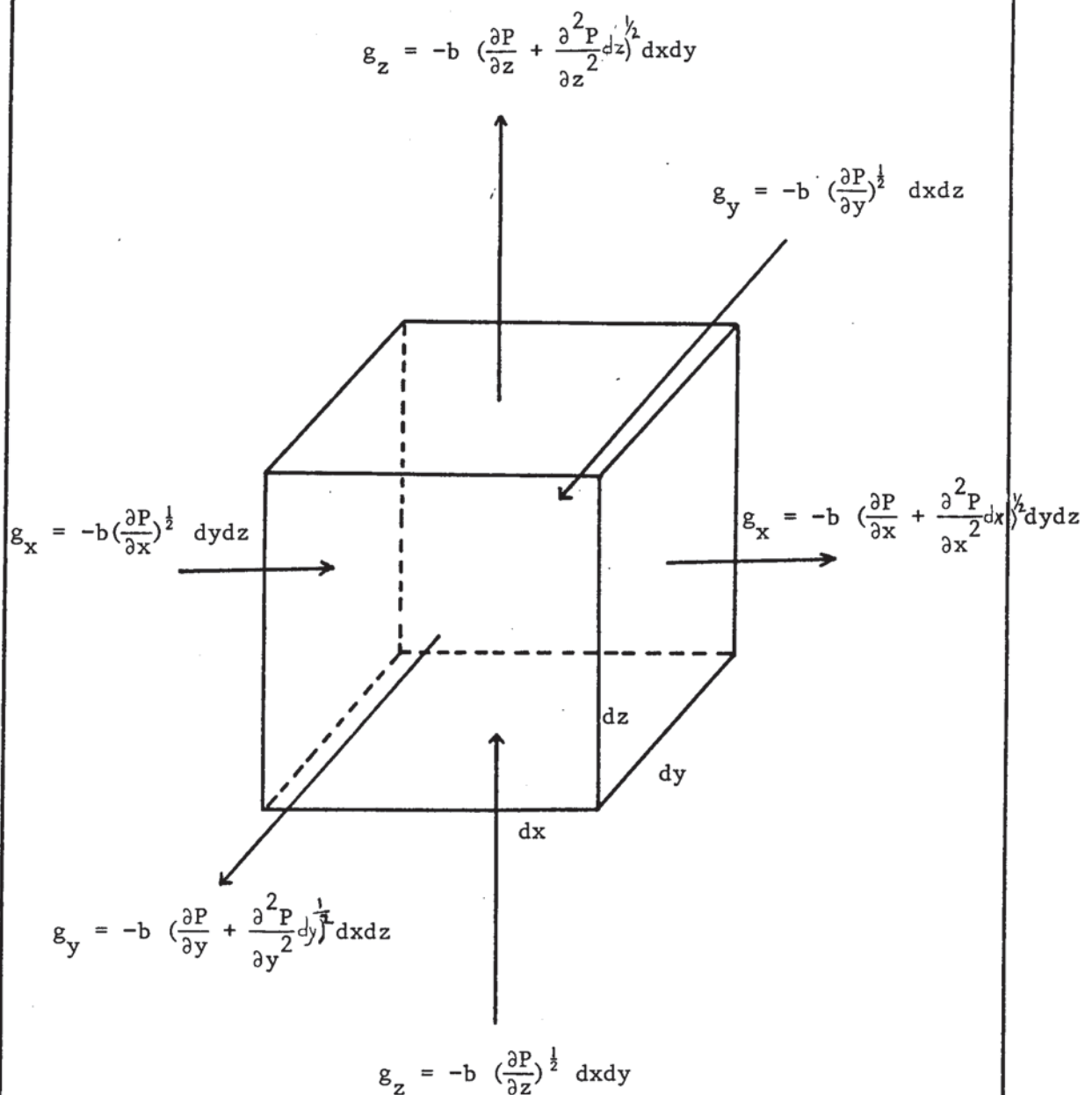


Figure 9.2 Mass Balance Over the  $(dx dy dz)$  Elemental Volume in Turbulent Gas Flow

### 9.1.2 Turbulent Flow

For totally turbulent gas flow, equations 9.1a, 9.1b and 9.1c are reduced to:

$$-\frac{\partial P}{\partial x} = \frac{B}{\rho} g_x^2 \quad \dots\dots (9.4a)$$

$$-\frac{\partial P}{\partial y} = \frac{B}{\rho} g_y^2 \quad \dots\dots (9.4b)$$

$$-\frac{\partial P}{\partial z} = \frac{B}{\rho} g_z^2 \quad \dots\dots (9.4c)$$

and

$$g_x = -b \left( \frac{\partial P}{\partial x} \right)^{\frac{1}{2}}$$

$$g_y = -b \left( \frac{\partial P}{\partial y} \right)^{\frac{1}{2}}$$

$$g_z = -b \left( \frac{\partial P}{\partial z} \right)^{\frac{1}{2}}$$

where  $b = \frac{\rho}{B}$

Gas mass flow rate out of the elemental volume in x, y and z directions is given respectively below, (see Fig. 9.2).

$$g_x = -b \left( \frac{\partial P}{\partial x} + \frac{\partial^2 P}{\partial x^2} dx \right)^{\frac{1}{2}} dydz \quad \dots\dots (9.5a)$$

$$g_y = -b \left( \frac{\partial P}{\partial y} + \frac{\partial^2 P}{\partial y^2} dy \right)^{\frac{1}{2}} dxdz \quad \dots\dots (9.5b)$$

$$g_z = -b \left( \frac{\partial P}{\partial z} + \frac{\partial^2 P}{\partial z^2} dz \right)^{\frac{1}{2}} dxdy \quad \dots\dots (9.5c)$$



To simplify these equations:

$$\begin{aligned}
 g_x &= -b \left( \frac{\partial P}{\partial x} + \frac{\partial^2 P}{\partial x^2} dx \right)^{\frac{1}{2}} dydz \\
 &= -b \left( \frac{\partial P}{\partial x} \right)^{\frac{1}{2}} \left( 1 + \frac{\partial^2 P / \partial x^2}{\partial P / \partial x} dx \right)^{\frac{1}{2}} dydz \\
 &= -b \left( \frac{\partial P}{\partial x} \right)^{\frac{1}{2}} \left( 1 + \frac{1}{2} \frac{\partial^2 P / \partial x^2}{\partial P / \partial x} dx \right) dydz
 \end{aligned}$$

and finally

$$g_x = -b \left( \frac{\partial P}{\partial x} \right)^{\frac{1}{2}} dydz - \frac{b}{2} \frac{\partial^2 P / \partial x^2}{\sqrt{\partial P / \partial x}} dx dydz \quad \dots (9.6a)$$

Similarly

$$g_y = -b \left( \frac{\partial P}{\partial y} \right)^{\frac{1}{2}} dx dz - \frac{b}{2} \frac{\partial^2 P / \partial y^2}{\sqrt{\partial P / \partial y}} dx dy dz \quad \dots (9.6b)$$

$$g_z = -b \left( \frac{\partial P}{\partial z} \right)^{\frac{1}{2}} dx dy - \frac{b}{2} \frac{\partial^2 P / \partial z^2}{\sqrt{\partial P / \partial z}} dx dy dz \quad \dots (9.6c)$$

Performing a mass balance across the elemental volume for incompressible gas where there is no accumulation or depletion will result in:

$$\begin{aligned}
 & \left[ -b \left( \frac{\partial P}{\partial x} \right)^{\frac{1}{2}} dydz + b \left( \frac{\partial P}{\partial x} \right)^{\frac{1}{2}} dydz + \frac{b}{2} \frac{\partial^2 P / \partial x^2}{\sqrt{\partial P / \partial x}} dx dy dz \right] + \\
 & \left[ -b \left( \frac{\partial P}{\partial y} \right)^{\frac{1}{2}} dx dz + b \left( \frac{\partial P}{\partial y} \right)^{\frac{1}{2}} dx dz + \frac{b}{2} \frac{\partial^2 P / \partial y^2}{\sqrt{\partial P / \partial y}} dx dy dz \right] + \\
 & \left[ -b \left( \frac{\partial P}{\partial z} \right)^{\frac{1}{2}} dx dy + b \left( \frac{\partial P}{\partial z} \right)^{\frac{1}{2}} dx dy + \frac{b}{2} \frac{\partial^2 P / \partial z^2}{\sqrt{\partial P / \partial z}} dx dy dz \right] = 0
 \end{aligned}$$

Which is reduced to

$$\frac{b}{2} \left[ \frac{\partial^2 P / \partial x^2}{\sqrt{\partial P / \partial x}} + \frac{\partial^2 P / \partial y^2}{\sqrt{\partial P / \partial y}} + \frac{\partial^2 P / \partial z^2}{\sqrt{\partial P / \partial z}} \right] dx dy dz = 0$$

$$\therefore \frac{\partial^2 P / \partial x^2}{\sqrt{\partial P / \partial x}} + \frac{\partial^2 P / \partial y^2}{\sqrt{\partial P / \partial y}} + \frac{\partial^2 P / \partial z^2}{\sqrt{\partial P / \partial z}} = 0 \quad \dots (9.7)$$

which is the mass balance equation for turbulent flow through the elemental volume.

### 9.1.3 Laminar and Turbulent Flows

For a laminar gas flow through some parts of the elemental volume and a turbulent gas flow through the other parts, the differential versions of the Ergun equation will contain both the laminar and the turbulent terms, and written as:

$$-\frac{\partial P}{\partial x} = \frac{A\mu}{\rho} g_x + \frac{B}{\rho} g_x^2$$

$$-\frac{\partial P}{\partial y} = \frac{A\mu}{\rho} g_y + \frac{B}{\rho} g_y^2$$

$$-\frac{\partial P}{\partial z} = \frac{A\mu}{\rho} g_z + \frac{B}{\rho} g_z^2$$

$g_x$ ,  $g_y$  and  $g_z$  are approximated as follows:

$$g_x = -\frac{\rho}{A\mu} \left( \frac{\partial P}{\partial x} \right) - \frac{\rho}{B} \left( \frac{\partial P}{\partial x} \right)^{\frac{1}{2}} \quad \dots (9.8a)$$

$$g_y = - \frac{\rho}{A\mu} \left( \frac{\partial P}{\partial y} \right) - \frac{\rho}{B} \left( \frac{\partial P}{\partial y} \right)^{\frac{1}{2}} \quad \dots\dots (9.8b)$$

$$g_z = - \frac{\rho}{A\mu} \left( \frac{\partial P}{\partial z} \right) - \frac{\rho}{B} \left( \frac{\partial P}{\partial z} \right)^{\frac{1}{2}} \quad \dots\dots (9.8c)$$

The continuity equation is given as:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \quad \dots\dots (9.9)$$

Where  $V_x$ ,  $V_y$ ,  $V_z$  are the velocities in the x, y and z directions respectively.

Since  $\rho$  is assumed to be constant, that is, incompressible fluid, then eqn. 9.9 can be rewritten as:

$$\frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} + \frac{\partial g_z}{\partial z} = 0 \quad \dots\dots (9.10)$$

From eqn. 9.8a

$$\begin{aligned} \frac{\partial g_x}{\partial x} &= \frac{\partial}{\partial x} \left[ -a \frac{\partial P}{\partial x} - b \left( \frac{\partial P}{\partial x} \right)^{\frac{1}{2}} \right] \\ &= -a \frac{\partial^2 P}{\partial x^2} - \frac{b}{2} \left( \frac{\partial P}{\partial x} \right)^{-\frac{1}{2}} \left( \frac{\partial^2 P}{\partial x^2} \right) \end{aligned}$$

$$\text{Thus } \frac{\partial g_x}{\partial x} = -a \left( \frac{\partial^2 P}{\partial x^2} \right) \left[ 1 + \frac{b}{2a} \left( \frac{\partial P}{\partial x} \right)^{-\frac{1}{2}} \right] \quad \dots\dots (9.11a)$$

Similarly

$$\frac{\partial g_y}{\partial y} = -a \left( \frac{\partial^2 P}{\partial y^2} \right) \left[ 1 + \frac{b}{2a} \left( \frac{\partial P}{\partial y} \right)^{-\frac{1}{2}} \right] \quad \dots\dots (9.11b)$$

$$\frac{\partial g_z}{\partial z} = -a \left( \frac{\partial^2 P}{\partial z^2} \right) \left[ 1 + \frac{b}{2a} \left( \frac{\partial P}{\partial z} \right)^{-\frac{1}{2}} \right] \quad \dots (9.11c)$$

Substituting eqns. 9.11a, 9.11b and 9.11c in eqn. 9.10:

$$\begin{aligned} & -a \left( \frac{\partial^2 P}{\partial x^2} \right) \left[ 1 + \frac{b}{2a} \left( \frac{\partial P}{\partial x} \right)^{-\frac{1}{2}} \right] + a \left( \frac{\partial^2 P}{\partial y^2} \right) \left[ 1 + \frac{b}{2a} \left( \frac{\partial P}{\partial y} \right)^{-\frac{1}{2}} \right] \\ & + a \left( \frac{\partial^2 P}{\partial z^2} \right) \left[ 1 + \frac{b}{2a} \left( \frac{\partial P}{\partial z} \right)^{-\frac{1}{2}} \right] = 0 \quad \dots (9.12) \end{aligned}$$

If the flow is highly laminar, i.e.  $a \gg b$ , then eqn. 9.12 will be reduced to the laminar gas flow eqn. 9.3; if the flow is highly turbulent,  $b \gg a$ , then it will be reduced to the turbulent gas flow eqn. 9.7.

To solve eqns. 9.3, 9.7 and 9.12, the initial conditions are required as well as the boundary conditions, which will depend on a model for translating the velocity of the gas below the bed into pressure within the bed. Up until now this problem has not been solved, but some progress has been made by means of the much simpler models described below.

## 9.2 Simplified Models of Gas Maldistribution Through Packed Beds

It is clear that the solution of the differential equations given above is very complicated and it is very difficult to demonstrate similarity between flow patterns for geometrically and dynamically similar beds.

However, this demonstration can be done by assuming that the ratio of gas velocity in different parts of the packed bed is the

same in the model and the full-scale.

Two simple theoretical models have been proposed for this purpose for a tangential feed situation with no gas distributor.

#### 9.2.1 Two Areas Model

Two geometrically and dynamically similar beds with scale ratio of  $R$  are shown in Fig. 9.3. The definitions of the symbols used in this model are given in Table 9.1.

As it was noticed in the last section that when the feed is introduced tangentially, there exist an annular part of the bed in which air always goes upwards at high velocities, whereas in the middle area the flow is downwards in short packed depths and upwards in deep beds.

Also, it was found that air below the bed has high velocities in the annular area and low velocities at the middle of the column. In the formulation of this model it is assumed that  $S_1$  is equivalent to that part of the packed bed immediately above the high velocity gas flow where the high pressure is generated, (see Section 8). This part is separated by an impermeable membrane from the middle part.

In this model, the criteria for positive, zero, and negative flow in the other part of the bed is obtained, as well as that for similarity of flow patterns in geometrically and dynamically similar beds, (defined by identical velocity ratios).

Note that the experimental work in Section 8 showed that the width of the high pressure annulus is approximately the same as the inlet pipe diameter, and that  $k$  is assumed to be the same for both beds.

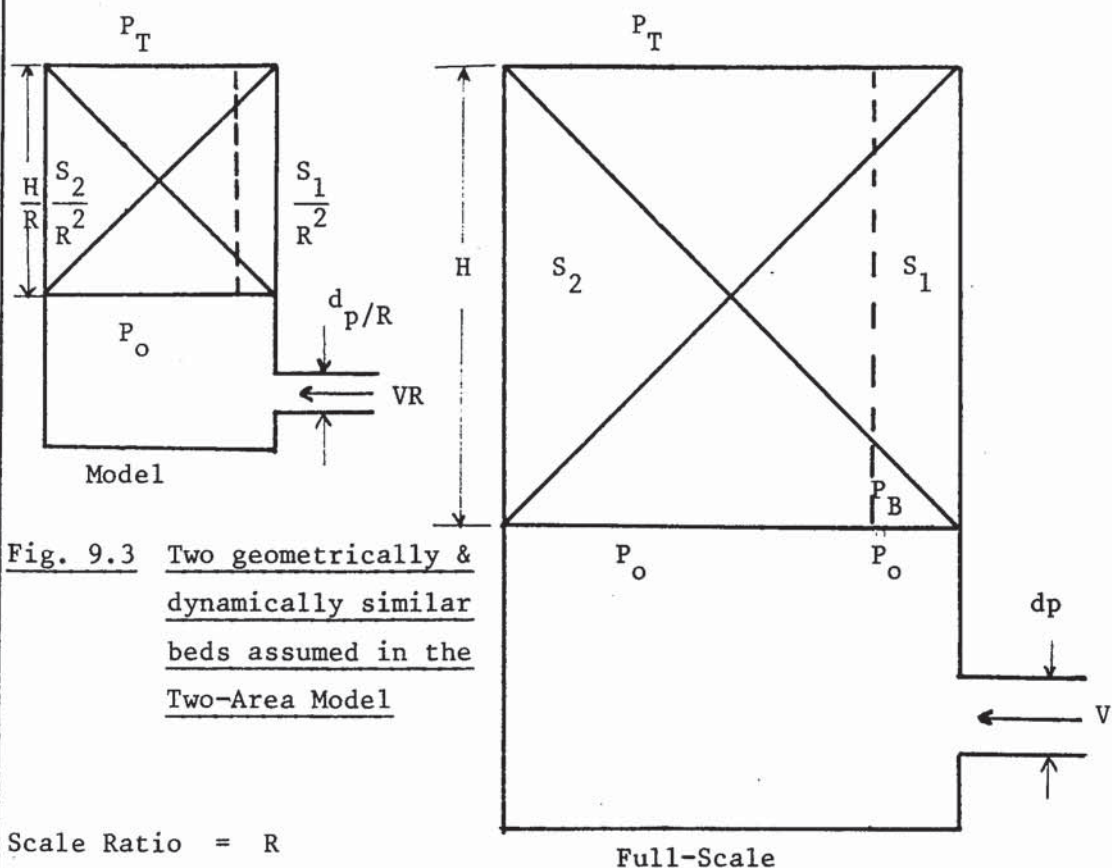


Table 9.1 Definition of the symbols used in Section 9

Property	Symbol for:	
	full-scale	Model
Packed Height	$H$	$H/R$
Packing size	$d$	$d/R$
Packing area/unit volume	$a$	$Ra$
Bed Diameter	$D$	$D/R$
Inlet Pipe Diameter	$dp$	$dp/R$
Inlet Pipe Velocity	$V$	$RV$
Up-flow Area	$S_1$	$S_1/R^2$
Up or Downflow Area	$S_2$	$S_2/R^2$
Up-Velocity in $S_1$	$U_1$	$RU_1$



Table 9.1 continued

Property	Symbol for:	
	Full-scale	Model
Velocity in $S_2$	$U_2$	$RU_2$
Drag Coefficient in Packed Bed	$C$	$C$
Pressure Generating Constant Below the Bed	$k$	$k$
Static Pressure at the Bottom of $S_1$	$P_{Bf}$	$P_{Bm}$
Pressure Drop across $S_1$	$\Delta P_{1f}$	$\Delta P_{1m}$
Pressure Drop across $S_2$	$\Delta P_{2f}$	$\Delta P_{2m}$
Gas Density	$\rho$	$\rho$
Pressure at the Top of the Bed	$P_T$	$P_T$
Static Pressure Below the Bed	$P_{of}$	$P_{om}$

For the full-scale bed, the pressure drop of the dry packing across  $S_1$ , (high pressure area), is given by

$$\Delta P_{1f} = C.(\rho U_1^2).H.a \quad \dots\dots (9.13)$$

For  $P_{of} = P_T$ , the pressure generated is proportional to  $(\rho V^2)$  inside the feed inlet, i.e.

$$P_{Bf} = k(\rho V^2) \quad \dots\dots (9.14)$$

For the model bed, the pressure drop of the dry packing across  $S_1/R_1^2$ , (high pressure area), is given by:

$$\begin{aligned} \Delta P_{1m} &= C [\rho(RU_1)^2] \cdot \frac{H}{R} \cdot Ra \\ &= R^2.C.H.a (\rho U_1^2) \quad \dots\dots (9.15) \end{aligned}$$

In both the model and the full scale beds, the flow of gas is upwards in the area of high pressure. The direction of the gas flow in the area of low pressure (middle area) may be:

a) Zero-Flow in  $S_2$ : This case happens in the full-scale bed when  $\Delta P_{1f} = P_{Bf}$ .

$$\text{i.e. } C.H.a (\rho U_1^2) = k(\rho V^2) \quad \dots\dots (9.16)$$

And in the model when  $\Delta P_{1m} = P_{Bm}$

$$\text{i.e. } R^2.C.H.a (\rho U_1^2) = k R^2(\rho V^2)$$

$$\therefore C.H.a (\rho U_1^2) = k (\rho V^2) \quad \dots\dots (9.16a)$$

Which is the same condition for zero flow in the low pressure area in the full-scale bed.

$$\therefore \boxed{C.H.a.U_1^2 = kV^2} \quad \dots\dots (9.17)$$

is the general equation for zero flow in the low pressure area in a packed bed.

An expression for zero flow in  $S_2$  may be obtained in terms of the cross-sectional area of the inlet pipe and the upflow area  $S_1$ ; that is:

$$S_1 U_1 = V a_p \quad \text{mass balance}$$

where  $a_p$  is the area of the feed pipe

$$\therefore U_1 = V \frac{a_p}{S_1}$$

Substitute for  $U_1$  in the general equation 9.17

$$C.H.a \left( \frac{a_p}{S_1} \right)^2 V^2 = kV^2$$

$$\therefore \frac{C.H.a}{k(S_1/a_p)^2} = 1 \quad \dots\dots (9.18)$$

eqn. 9.18 represents the critical condition.

b) Non-Zero Flow: In this case the gas is flowing either upwards or downwards in the low pressure area  $S_2$ .

In general

$$P_B = k(\rho V^2) + P_O \quad \dots\dots (9.19a)$$

$$\Delta P_1 = (P_B - P_T) = C.H.a (\rho U_1^2) \quad \dots\dots (9.19b)$$

$$\Delta P_2 = (P_O - P_T) \text{ or } (P_T - P_O) \quad \dots\dots (9.19c)$$

Where  $P_O$ ,  $P_B$ ,  $\Delta P_1$  and  $\Delta P_2$  are the initial pressure below the bed, the static pressure at the bottom of the bed, pressure drop across  $S_1$  and pressure drop across  $S_2$  respectively.

Substituting for  $P_B$  from 9.19a into 9.19b

$$\therefore \Delta P_1 = (P_O - P_T) + k(\rho V^2) = C.H.a(\rho U_1^2)$$

$$\therefore \boxed{(P_O - P_T) = C.H.a(\rho U_1^2) - k(\rho V^2)} \quad \dots\dots (9.20)$$

which is the general equation for upwards or downwards flow in  $S_2$ ; that is:

$$\text{if } C.H.a(\rho U_1^2) > k(\rho V^2)$$

$$\therefore (P_O - P_T) \text{ is +ve and the flow is upwards in } S_2,$$

$$\text{and if } C.H.a(\rho U_1^2) < k(\rho V^2)$$

$$\therefore (P_O - P_T) \text{ is -ve and the flow is downwards in } S_2.$$

It is of interest to develop the model further to show the expected effect of inlet pipe diameter on the gas flow pattern.

It was found experimentally, (see Section 7), that the width of the high pressure area is almost equal to the feed pipe diameter

$$\begin{aligned}
 S_1 &= \frac{\pi}{4} [ D^2 - (D - 2dp)^2 ] \\
 &= \frac{\pi}{4} [ D^2 - (D^2 - 4 Ddp + 4dp^2) ] \\
 &= \frac{\pi}{4} ( 4Ddp - 4dp^2 )
 \end{aligned}$$

$$\therefore S_1 = \pi(Ddp - dp^2)$$

$$a_p = \pi \frac{dp^2}{4}$$

$$\text{then } \frac{S_1}{a_p} = 4 \left( \frac{D}{dp} - 1 \right) \quad \dots (9.21)$$

In general, gas will flow either upwards or downwards in the lower pressure area of the bed,  $S_2$ . The similarity between the flow patterns in the full-scale and the model may be demonstrated by showing that the velocity ratio  $\frac{U_1}{U_2}$  is the same in both the model and the full-scale.

The equations derived below all show the effect of the ratio of the inlet pipe diameter to column diameter on the  $\frac{U_2}{U_1}$  ratio; that is on gas maldistribution. To perform this, each case is studied separately as follows:

#### b.1 Gas Flows Upwards in $S_2$

From eqn. 9.20

$$(P_o - P_T) = C.H.a (\rho U_1^2) - k (\rho V^2) \text{ pressure drop across } S_1$$

From eqn. 9.19c

$$\Delta P_2 = P_o - P_T = C.H.a(\rho U_2^2) \quad \text{pressure drop across } S_2$$

$$\therefore C.H.a(\rho U_1^2) + k(\rho V^2) = C.H.a(\rho U_2^2)$$

$$C.H.a(U_1^2 - U_2^2) = kV^2$$

$$\therefore (U_1^2 - U_2^2) = \frac{kV^2}{C.H.a}$$

$$U_1^2 \left[ 1 - \left( \frac{U_2}{U_1} \right)^2 \right] = \frac{kV^2}{C.H.a}$$

$$\therefore \left( \frac{U_2}{U_1} \right)^2 = 1 - \frac{k}{C.H.a} \left( \frac{V}{U_1} \right)^2 \quad \dots\dots (9.22)$$

$\frac{U_2}{U_1}$  defines the gas maldistribution and flow pattern in the packed bed.

To get eqn. 9.22 in terms of  $\left( \frac{U_2}{U_1} \right)$  only:

$$S_2 = \frac{\pi}{4} D^2 - S_1$$

$$S_1 = \pi (Ddp - dp^2)$$

$$\therefore S_2 = \frac{\pi}{4} D^2 - \pi (Ddp - dp^2)$$

$$\therefore \frac{S_2}{S_1} = \frac{\left( \frac{D}{dp} \right)^2}{4 \left( \frac{D}{dp} - 1 \right)^2} \quad \dots\dots (9.23)$$

Mass balance gives

$$S_1 U_1 + S_2 U_2 = a_p V$$

$$1 + \frac{S_2 U_2}{S_1 U_1} = \frac{a_p}{S_1} \left( \frac{V}{U_1} \right)$$

$$1 + \left( \frac{S_2}{S_1} \right) \left( \frac{U_2}{U_1} \right) = \frac{1}{S_1/a_p} \left( \frac{V}{U_1} \right)$$



Substitute for  $\frac{S_2}{S_1}$  from eqn. 9.23 and  $S_1/a_p$  from 9.21 in the above equation:

$$\therefore 1 + \left[ \frac{\left(\frac{D}{dp}\right)^2}{4 \left(\frac{D}{dp} - 1\right)} - 1 \right] \left( \frac{U_2}{U_1} \right) = \frac{1}{4 \left(\frac{D}{dp} - 1\right)} \left( \frac{V}{U_1} \right)$$

$$\text{Let } A = \frac{\left(\frac{D}{dp}\right)^2}{4 \left(\frac{D}{dp} - 1\right)} - 1, \quad B = 4 \left(\frac{D}{dp} - 1\right)$$

$$\therefore 1 + A \left( \frac{U_2}{U_1} \right) = \frac{1}{B} \left( \frac{V}{U_1} \right)$$

$$\therefore \left( \frac{V}{U_1} \right) = B + AB \left( \frac{U_2}{U_1} \right)$$

$$\text{and} \quad \left( \frac{V}{U_1} \right)^2 = [B + AB \frac{U_2}{U_1}]^2 \quad \dots\dots (9.24)$$

Substitute for  $\left( \frac{V}{U_1} \right)^2$  from 9.24 into 9.22:

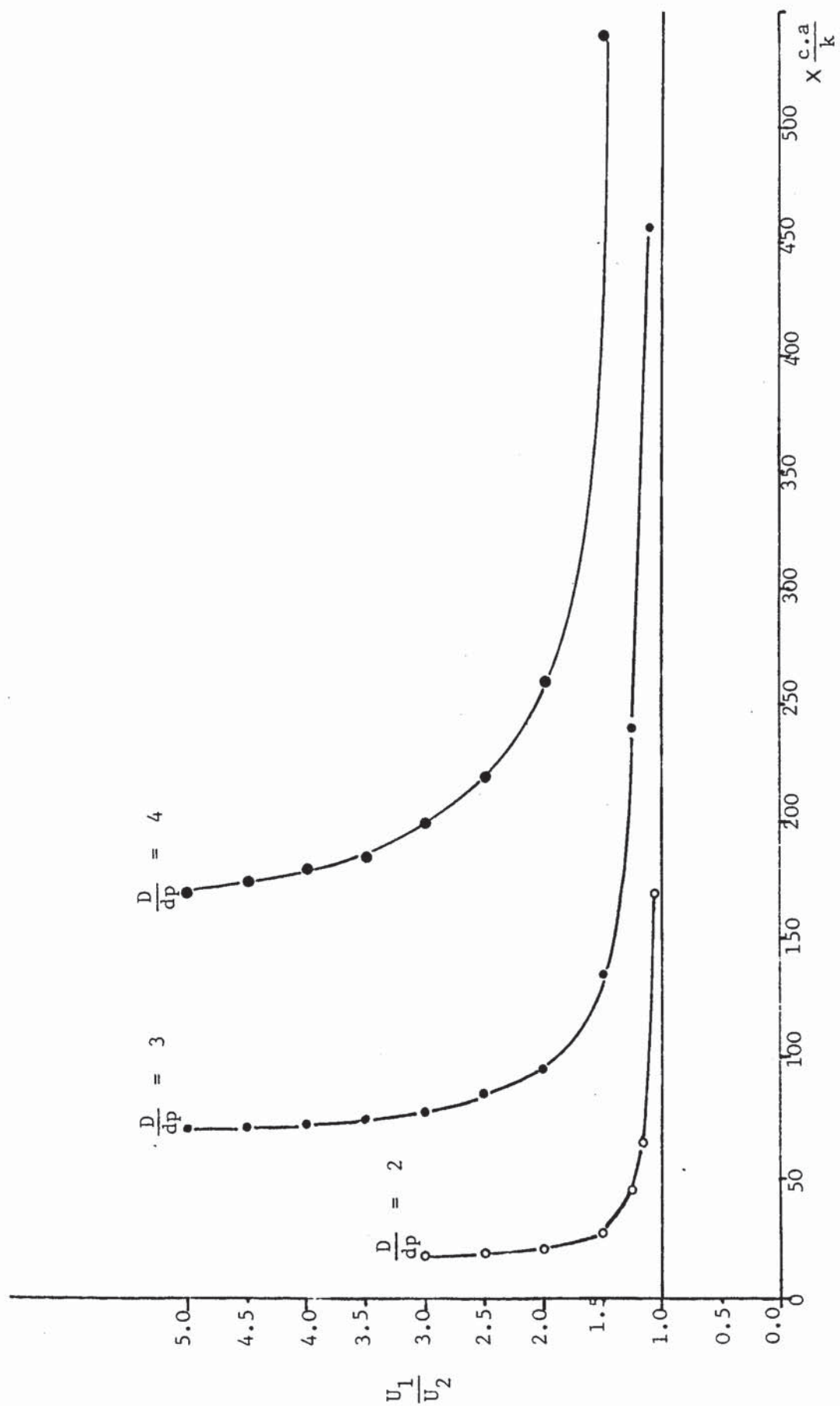
$$\left( \frac{U_2}{U_1} \right)^2 = 1 - \frac{k}{C.H.a} [B + AB \frac{U_2}{U_1}]^2$$

$$AB = \left[ \frac{\left(\frac{D}{dp}\right)^2}{4 \left(\frac{D}{dp} - 1\right)} - 1 \right] 4 \left(\frac{D}{dp} - 1\right)$$

$$= \left(\frac{D}{dp}\right)^2 - 4 \left(\frac{D}{dp} - 1\right)$$

$$= \left(\frac{D}{dp}\right)^2 - 4 \frac{D}{dp} + 4 = \left(\frac{D}{dp} - 2\right)^2$$

$$\therefore \left( \frac{U_2}{U_1} \right)^2 = 1 - \frac{k}{C.H.a} [4 \left(\frac{D}{dp} - 1\right) + \left(\frac{D}{dp} - 2\right)^2 \left( \frac{U_2}{U_1} \right)]^2 \dots (9.25)$$



Graph 9.1 Variation of  $\frac{U_1}{U_2}$  with P.H. at different  $\frac{D}{dp}$  ratios

This flow pattern function evaluates the packed height required for  $\frac{U_2}{U_1} \rightarrow 1$  for different  $\frac{D}{d_p}$  ratios which describes the effect of inlet pipe diameter on gas maldistribution as shown in Graph 9.1.

Since  $\left( \frac{k}{C.H.a} \right)$  is the same for geometrically similar beds at same Reynolds number

$$\therefore \frac{U_2}{U_1} = f \left( \frac{D}{d_p}, \frac{k}{C.H.a} \right) \text{ only}$$

#### b.2 Gas Flow Downwards in $S_2$

In this case, the pressure drop across  $S_2$  is given by

$$\Delta P_2 = P_T - P_O = C.H.a (\rho U_2^2)$$

And pressure drop across  $S_1$  is given by

$$\Delta P_1 = P_O + k (\rho V^2) - P_T = C.H.a (\rho U_1^2)$$

$$P_O - P_T = C.H.a (\rho U_1^2) - k(\rho V^2)$$

$$\therefore C.H.a (U_1^2 + U_2^2) = kV^2$$

$$\text{and } \left( \frac{U_2}{U_1} \right)^2 = \frac{k}{C.H.a} \left( \frac{V}{U_1} \right)^2 - 1 \quad \dots\dots (9.26)$$

Air flows upwards in  $S_1$  at a rate greater than the feed, so air is flowing downwards in  $S_2$  to maintain the mass balance.

The mass balance equation is given by

$$S_1 U_1 - S_2 U_2 = a_p V$$

$$1 - \left( \frac{S_2}{S_1} \right) \left( \frac{U_2}{U_1} \right) = \frac{1}{S_1/a_p} \left( \frac{V}{U_1} \right)$$

Substituting for  $\frac{S_2}{S_1}$  from eqn. 9.23 and  $\frac{S_1}{a_p}$  from 9.21 in the above equation

$$1 - \left[ \frac{\left(\frac{D}{dp}\right)^2}{4 \left(\frac{D}{dp} - 1\right)} - 1 \right] \left( \frac{U_2}{U_1} \right) = \frac{1}{4 \left(\frac{D}{dp} - 1\right)} \left( \frac{V}{U_1} \right)$$

$$1 - A \left( \frac{U_2}{U_1} \right) = \frac{1}{B} \left( \frac{V}{U_1} \right)$$

$$\therefore \left( \frac{V}{U_1} \right)^2 = [B - AB \left( \frac{U_2}{U_1} \right)]^2 \quad \dots\dots (9.27)$$

eqn. 9.27 in eqn. 9.26

$$\therefore \left( \frac{U_2}{U_1} \right)^2 = \frac{k}{C.H.a} \left[ 4 \left( \frac{D}{dp} - 1 \right) - \left( \frac{D}{dp} - 2 \right)^2 \frac{U_2}{U_1} \right]^2 - 1 \quad \dots\dots (9.28)$$

which again shows that  $\frac{U_2}{U_1} = f_1 \left( \frac{D}{dp}, \frac{k}{C.H.a} \right)$  only

Both equations 9.25 and 9.28, if  $\frac{U_2}{U_1} = 0$ , i.e.  $U_2 = 0$ , will be reduced to

$$\frac{C.H.a}{k \left[ 4 \left( \frac{D}{dp} - 1 \right) \right]^2} = 1 \quad \dots\dots (9.29)$$

$\therefore$  For larger feed inlet diameter,  $dp$ , i.e. smaller  $\frac{D}{dp}$  ratio, and larger packed height,  $H$ , the left hand side of equation 9.29 will become greater than 1. This means that the gas is flowing upwards in  $S_2$ , which explains the uniform flow in beds of small diameter and large packed height.

But in case of shallow large diameter packed beds, where pipe inlet diameter is small relative to bed diameter and packed depth is small, the left hand side of eqn. (9.29) will become less

than 1 and the downwards flow in the middle of the bed appears, giving rise to severe gas maldistribution.

Thus the designs of packed beds in which the left hand side of eqn. 9.29 less than one could be identified as "Designs at Risk" of gas maldistribution.

### 9.2.2 Cross-Flow Model

In this model it is assumed that the high pressure region,  $S_1$ , is connected at the bottom of the bed to the low pressure region,  $S_2$ , by a channel which permits the gas to flow across. This addition of cross-flow provides a more realistic description of the expected flow pattern. The gas can move radially through this channel. Fig. 9.4 shows cross sections in two geometrically and dynamically similar packed beds with a scale ratio  $R$ , where:

$\Delta P_c$  is the pressure drop due to the cross-flow

$S_c$  is the cross sectional area of the channel.

In this model, the following assumptions were made

(1) The length of the channel,  $z$ , is proportional to  $\frac{1}{R}$ , but  $z$  is infinitely small, that is:

$$S_1 + S_2 = \frac{\pi}{4} D^2$$

(2) The width of the high pressure region  $S_1$  is the same as the feed pipe diameter.

(3)  $k$  is the same for both the model and full-scale packed beds.

(4) The gas flow direction is:

(a) Always upwards in  $S_1$ .

(b) Zero-flow, upwards, or downwards in  $S_2$

Similar to the two areas model, this model deals with the situations of zero-flow, upwards flow, and downwards flow in  $S_2$  as

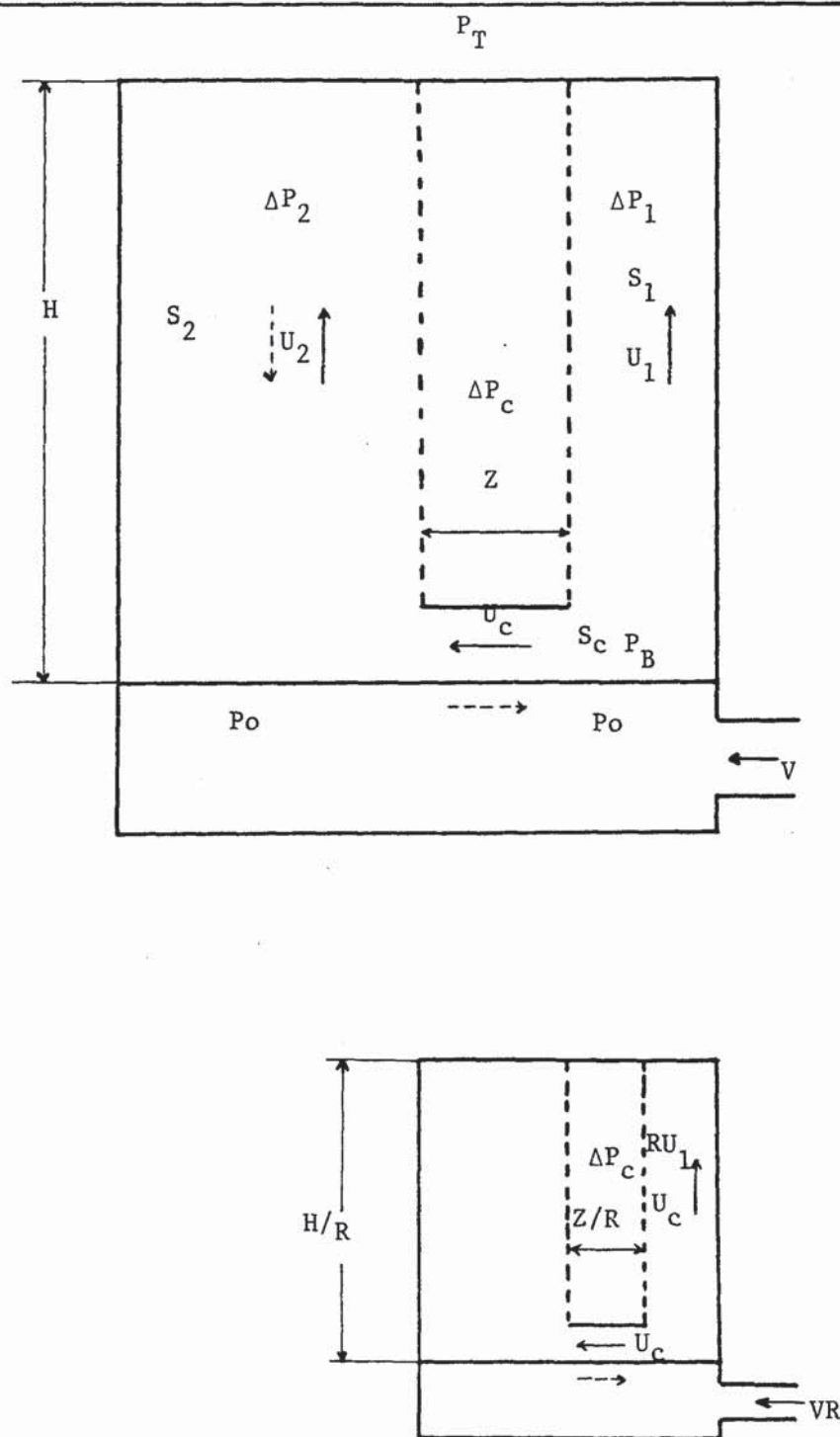


Fig. 9.4 Two Geometrically and Dynamically Similar  
Beds Assumed in the Cross-flow Model



follows:

(a) Zero-Flow in  $S_2$

In this case there is no flow in  $S_2$  which means that all the gas flowing across the channel from  $S_1$  to  $S_2$  is recycled to  $S_1$  from beneath the bed, and that

$$P_o = P_T \quad \text{..... (9.30)}$$

For the geometrically and dynamically similar beds shown in Fig. 9.4, the flow equation for zero-flow will be derived as follows:

(1) For the Full-scale bed:

$$\begin{aligned} \Delta P_{cf} &= P_{Bf} - P_{of} \\ P_{Bf} &= k(\rho V^2) + P_{of} \\ \therefore P_{cf} &= k(\rho V^2) \\ \text{c.z.a } (\rho U_c^2) &= k(\rho V^2) \\ \therefore U_c^2 &= \frac{k}{\text{c.z.a}} V^2 \quad \text{..... (9.31)} \end{aligned}$$

(2) For the Model bed:

$$\begin{aligned} \Delta P_{cm} &= P_{Bm} - P_{om} \\ P_{Bm} &= k.R^2.(\rho V^2) + P_{om} \\ \therefore P_{cm} &= k.R^2.(\rho V^2) \\ \text{c. } \frac{z}{R}.a.R.R^2(\rho U_c^2) &= k.R^2.(\rho V^2) \\ \therefore U_c^2 &= \frac{k}{\text{c.z.a}} V^2 \quad \text{..... (9.32)} \end{aligned}$$

which is identical to eqn. 9.31. This proves the validity of the principle of dynamic similarity.

(b) Non-zero Flow

In this case the gas flowing upwards or downwards in  $S_2$ .

In general:

$$\Delta P_1 = P_B - P_T \quad \dots\dots (9.33a)$$

$$\Delta P_c = P_B - P_o \quad \dots\dots (9.33b)$$

$$\Delta P_2 = (P_o - P_T) \text{ or } (P_T - P_o) \quad \dots\dots (9.33c)$$

$$\therefore \Delta P_2 = \Delta P_1 - \Delta P_c \quad \dots\dots (9.34)$$

Eqn. 9.34 is the general equation of pressure drops in non-zero flow in  $S_2$ . If  $\Delta P_1 > \Delta P_c$ , then the flow is upwards in  $S_2$ ; if  $\Delta P_1 < \Delta P_c$ , then the flow is downwards in  $S_2$ .

Substituting  $P_B = k(\rho V^2) + P_o$  in eqn. 9.33b:

$$\Delta P_c = k(\rho V^2)$$

$$\text{c.z.a } (\rho U_c^2) = k(\rho V^2)$$

$$\therefore \boxed{U_c^2 = \frac{k}{\text{c.z.a}} V^2} \quad \dots\dots (9.35)$$

Eqn. 9.35 is identical to eqn. 9.31 in zero-flow situation. Thus it could be concluded that  $U_c$  is proportional to the velocity in the feed pipe,  $V$ , and that, at a constant  $V$ , the gas passing through the channel has a constant velocity,  $U_c$  from  $S_1$  to  $S_2$  whether or not there was a flow in  $S_2$ .

The upwards and downwards flows are going to be studied below:

b.1 Upward flow in  $S_2$

$$\text{From eqn. 9.34: } \Delta P_2 = \Delta P_1 - \Delta P_c$$

$$\therefore \text{C.H.a} (U_2^2) = \text{C.H.a} (U_1^2) - \text{C.z.a} (U_c^2)$$

Substituting for  $(U_c^2)$  from eqn. (9.35) gives:

$$\text{C.H.a} (U_2^2) = \text{C.H.a} (U_1^2) - k(V^2)$$

Dividing by  $\text{C.H.a} (U_1^2)$  gives:

$$\left( \frac{U_2}{U_1} \right)^2 = 1 - \frac{k}{\text{C.H.a}} \left( \frac{V}{U_1} \right)^2 \quad \dots (9.36)$$

Eqn. 9.36 is identical to eqn. 9.22 obtained for upwards flow in the "Two Areas" model. It implies that the cross-flow is simply recycled around the areas at the bottom of the bed with a high velocity generated pressure at those areas where the pressure is the static pressure under the bed.

The mass balance equation will be:

$$a_p V = S_1 U_1 + S_2 U_2 \quad \dots (9.37)$$

Therefore, eqn. 9.36 can be written in terms of  $\left( \frac{U_2}{U_1} \right)$  and  $\left( \frac{D}{dp} \right)$  by the same method followed in the "Two Areas" model as:

$$\left( \frac{U_2}{U_1} \right)^2 = 1 - \frac{k}{\text{C.H.a}} \left[ 4 \left( \frac{D}{dp} - 1 \right) + \left( \frac{D}{dp} - 2 \right)^2 \left( \frac{U_2}{U_1} \right) \right]^2 \quad \dots (9.38)$$

which is identical to eqn. 9.25.

This flow pattern equation evaluates the packed height required for  $\frac{U_2}{U_1} \rightarrow 1$  for different  $\frac{D}{dp}$  values in an identical way to that shown in Graph 9.1.

## b.2 Downwards flow in $S_2$

In this case the gas is flowing downwards in  $S_2$ , which means that  $P_T > P_O$ , and equation 9.34 will be written as:

$$\Delta P_2 = \Delta P_c - \Delta P_1$$

By a similar method to that followed in the upwards flow, the flow pattern equation for downwards flow can be obtained as:

$$\left( \frac{U_2}{U_1} \right)^2 = \frac{k}{C.H.a} \left[ 4 \left( \frac{D}{dp} - 1 \right) - \left( \frac{D}{dp} - 2 \right)^2 \left( \frac{U_2}{U_1} \right) \right]^2 - 1 \quad \dots (9.39)$$

Taking into consideration that the mass balance equation is:

$$a_p V = S_1 U_1 - S_2 U_2 \quad \dots (9.40)$$

Eqn. 9.39 is identical to eqn. 9.28 produced in the "Two Area" model for downwards flow in  $S_2$ .

Both eqns. 9.38 and 9.39 are functions of  $\left( \frac{U_2}{U_1} \right)$  and  $\left( \frac{k}{C.H.a} \right)$  only. And if  $\frac{U_2}{U_1} = 0$ , both equations will be reduced to:

$$\frac{C.H.a}{k \left[ 4 \left( \frac{D}{dp} - 1 \right) \right]^2} = 1 \quad \dots (9.41)$$

Which is identical to eqn. 9.29 results in the previous model, and represents the critical conditions. The packed beds with designs "at risk" of gas maldistribution could be identified as those designs at which the left hand side of eqn. 9.41 is less than 1.

### 9.3 Discussion of the Simplified Theoretical Models

The two simplified theoretical models proposed above, the Two Areas model and the Cross-Flow model produce the same flow pattern function, given in eqns. 9.25 and 9.38, which evaluates the packed height required for uniform distribution at different  $\frac{D}{dp}$

ratios. This function is solved and presented in Graph 9.1 which shows that the variation of  $\frac{U_1}{U_2}$  with packed height is similar to the variation of  $\phi$  with packed height observed in previous sections of this work. The graph shows the importance of the ratio of tower diameter to feed pipe diameter. A negligible packed depth is required to obtain a uniform distribution for  $\frac{D}{dp} = 2$ , but the depth required increases rapidly and it is quite large for  $\frac{D}{dp} = 3$ .

Also, the packed beds designs which are exposed to gas maldistribution, "Designs at Risk", could be identified from the general equation of the critical conditions which is produced by the two simple models as:

$$\frac{C.H.a}{k \left[ 4 \left( \frac{D}{dp} - 1 \right) \right]^2} \leq 1$$

From which it could be seen clearly that the large diameter packed beds with relatively small packed height and vapour inlet diameter are more likely to be maldistributed than the standard packed beds in which the packed height is much larger than the diameter of the bed, and the vapour inlet is relatively large.

One interesting result is that both simplified theoretical models one with cross-flow and one without gave the same equations. For the flow pattern  $\left( \frac{U_1}{U_2} \right)$ , equations 9.25 and 9.38, and for the condition which determines a negative down flow in the bed, eqns. 9.28 and 9.40. A physical explanation of this is that the cross-flow gas simply recirculates at a constant rate between high pressure and low pressure regions in the bed. The rate of recirculation is proportional to the inlet pipe velocity, but is independent of the height of packing. It is evident that the

cross-flow model needs to be developed further to permit evaluation of the effect of cross flow at different distances above the bottom of the bed.

#### 9.4      Conclusions

Three theoretical models of maldistributed gas flow in packed beds were proposed, these are; the three dimensional gas flow, the two areas model, and the cross-flow model. The last two models, which are simplified theoretical models could be solved and packed beds designs "at Risk" of maldistribution could be identified and the principle of dynamic similarity could be proved.



## SECTION X

### TESTING MODELS OF GAS DISTRIBUTORS USED

#### IN EXISTING LARGE DIAMETER COLUMNS

##### Introduction

In this section of the work a pump around packed bed in the vacuum crude fractionation tower, described in Section 2, is chosen as an example of an "at risk" design. This pump around packed bed has all the "at risk" requirements of gas maldistribution; large diameter, (7.3m), short packed height, (1 to 2m), and relatively small vapour inlet, (usually less than 1m in diameter).

The experimental work in this section deals with the test of existing gas distribution devices of the type used in large diameter vacuum crude fractionation columns.

In Section 6 of this work, the rules of scale-up of packed beds were obtained experimentally. Those rules are going to be applied in the construction of the model used in this section.

It should be noted, (see Section 2.2), that at the present, the designers of the pump around sections regard the Internal Cylinder and the annular space as a gas distributor. As shown below, the internal cylinder does little to improve the distribution of the gas. In the full-scale column, which was the prototype for this work, most improvement of the gas distribution was obtained from the particular design of the draw-off tray which was used.

## 10.1 Construction of the Model

The laboratory study uses a scaled down model exactly geometrically and dynamically similar to the full-scale pump around section of a 7.3m diameter vacuum crude fractionation column but  $\frac{1}{6}$  the size. A complete set of drawings of the full-scale column were obtained and used to provide the exactly scaled-down dimensions of the model.

### 10.1.1 The choice of the $\frac{1}{6}$ scale-down ratio

The  $\frac{1}{6}$  scaling ratio was used because it is only possible to obtain commercially available Pall ring packings over a size range of ( $\frac{5}{8}$  in to  $3\frac{1}{2}$  ins), which means that the smallest packing is about  $\frac{1}{6}$  the largest size. Thus a  $\frac{1}{6}$  scale model packed with 16 mm, ( $\frac{5}{8}$  in), packing is equivalent to a full scale column packed with ( $3\frac{1}{2}$  in) packing.

The shell of the 122 cm inside diameter column, described in Section 8.1, and shown in Plate 8.1, is used in this work. The internals, (the draw-off tray, the support plate, the support beam and the internal cylinder), were designed and are described below.

### 10.1.2 The draw-off tray

The draw-off tray is used in the full-scale fractionator to separate the liquid to be drawn off from the rising vapour. In the model, a 121 cm diameter draw-off tray is designed geometrically similar to the full-scale draw-off tray and constructed from 1.5 mm thick galvanized iron. It consists of caps and troughs, slots and a sump, (see Plate 10.1).

The caps, (see Fig. 10.2), are closed at one end and slightly tilted to direct the trapped liquid to the sump. Each cap is fitted to the troughs by means of two slotted legs to allow

for the adjustment of the cap height above the troughs. The troughs, (see Fig. 10.1), are designed to leave slots between them to allow for the rising gas to pass through. The troughs are opened to the sump and connected to each other at the middle by channels. The sump, (see Fig. 10.4), is 10 cm deep.

The draw-off tray is held by a 1 cm wide and 0.5 cm thick support ring which is fitted to the column shell. The dimensions of the draw-off tray are shown in Fig. 10.3.

#### 10.1.3 The Support Plate and the Lattice Support Beam

The packing support plate, (gas injection type), is constructed from an expanded metal plate and designed to combine the mechanical strength and convenience in installation. It is furnished in 2-piece construction, (see Plate 10.1). The dimensions of this 121 cm diameter plate are shown in Fig. 10.5.

The lattice support beam, (see Plate 10.1), is designed to support the weight of the packed bed and the column internals. It is constructed from 1 cm aluminium angle bar. The design specifications and dimensions are shown in Fig. 10.6.

#### 10.1.4 The Internal Cylinder Distributor

This distributor consists of an ordinary cylinder 90 cm in diameter opened at both ends and welded at the top end to a 121 cm outside diameter flange which fits the inside diameter of the column. This design of the distributor causes the gas feed to swirl in the annular gap formed between the internal cylinder and the column wall which is closed at the top. Thus the swirling gas emerges from the annulus and flows upwards inside the internal cylinder.

The general layout of the assembled column is shown in Fig. 10.7.

## 10.2 Testing the Internal Cylinder Gas Distributor

The quality of the existing design of the internal cylinder was experimentally investigated. It is shown that this design does very little to improve the distribution of the gas, and that most of the gas distribution improvement was a result of the draw-off tray. The work proceeded to test the draw-off tray as a gas distributor.

### 10.2.1 Experimental Procedure

The quality of a gas feed distributor is evaluated according to the proposed method given in Section 6, that is, the maldistribution factor  $\phi$  is plotted against the packed depth. The best distributor is that one which causes a uniform gas distribution, ( $\phi = \phi_{\min}$ ), at the shortest packed depth.

The minimum number of point velocity measurements required is evaluated by a method similar to that described in Section 6 and found to be 200 measurements equally distributed in a plane 3.5 cm above a bed randomly packed with 16 mm plastic pall rings. The configuration of the points, at which the air velocity is measured, is shown in Figures 10.8 and 10.9 for radial and tangential feed inlets respectively.

The modified hot-wire anemometer, described in Section 4 is used to measure the point velocities. The air is pumped to the column through a 15.2 cm in diameter feed nozzle.

The maldistribution factor  $\phi$  is obtained for a series of deep packed depths, ( 20 cm to 80 cm) and the minimum value of the maldistribution factor is found ( $\phi_{\min} \approx 19$ ).

### 10.2.2 Experimental Results

The point velocities were measured above a series of shallow packed depths, (6-36 cm), which are equivalent to (0.36-2.15m) in the real life pump around packed bed.

A complete set of experimental observations is given in Appendix A5.

The analysed results are summarized in Table 10.1 and presented below for the following situations:

#### 1) Radial Feed with No Distributor

$\phi$  is drawn against packed depth in Graph 10.1 and the gas distribution response surfaces are shown in Figs. 10.10a to 10.10d.

The results show a sharp decrease in  $\phi$  values with the packed depth increase at very short depths, this variation eases at higher packed depths. At a packed depth of 36 cm,  $\phi$  converges with  $\phi_{\min}$ ; that is more than 2m packed depth in the real pump-around packed bed.

#### 2) Radial Feed with the Draw-off Tray Fixed

The air is pumped radially immediately below the draw off tray whose slots were perpendicular to the feed jet direction as shown in Fig. 10.7.

The results are shown in Graph 10.2 and Figs. 10.11a to 10.11d, which show that  $\phi$  approaches  $\phi_{\min}$  at a packed depth of about 20 cm, that is about 1.2m depth in the full-scale pump-around packed bed.



### 3) Tangential Feed with No Distribution

The results, presented in Graph 10.3 and Figs. 10.12a and 10.12b, show a negative flow of the air at the middle area of the packed bed and high air velocities near the wall. This negative flow was observed in a bed packed to a depth of 12 cm which is equivalent to more than 0.7m in the full-scale packed bed. At a depth of 18 cm, this negative flow has disappeared but still very small air point velocities were obtained at the middle area. A depth of 30 cm, (equivalent to 1.8m in the full-scale bed) was not enough to bring the maldistribution factor to its minimum value. This implies the need for the gas distributor.

### 4) Tangential Feed with the Draw-off Tray Fixed

The processed results are shown in Graph 10.4 and Figs. 10.13a and 10.13b. No negative air flows were detected and  $\phi$  converges with  $\phi_{\min}$  at a depth of 24 cm, which is equivalent to about 1.5 m in the full-scale bed.

### 5) Tangential Feed with Internal Cylinder Gas Distributor

The results, presented in Graph 10.4 and Fig. 10.14, show the existence of negative air flows in the middle area in beds packed up to 12 cm, that is equivalent to 0.7m in the full-scale packed bed.

## 10.3 Discussion of the Results

The most important result of this part of the work is the observation of a negative, (downwards), gas flow for a tangential feed inlet and a shallow bed at the middle area. This phenomenon was observed when no distributor is used which reveals the importance of the use of gas distributors. The internal cylinder is used as a device to improve the distribution of the gas. As the



work shows that the internal cylinder causes very little improvement where the negative flow phenomenon exists both with and without the internal cylinder, a packed depth of 30 cm, (that is equivalent to 1.8m in the full-scale bed), was not enough to obtain the minimum gas maldistribution, (see Graph 10.5).

Another interesting result was obtained from this work, that is in both cases of radial and tangential feed inlets, a relatively short packed depth is required to bring down the value of  $\phi$  to its minimum when the draw-off tray is used. For a radial feed,  $\phi$  is brought down to about  $1.6 \phi_{\min}$  within a packed depth as short as 6 cm only which is equivalent to 36 cm in the full-scale bed; in the case when the feed is introduced tangentially,  $\phi$  is brought down to about  $1.45 \phi_{\min}$  within a depth of 12 cm only, that is about 0.7m in the full-scale bed.

This implies that this special design of the draw-off tray acts as a good distributor. Probably this is because:

- 1) The free area provided by this design of draw-off tray is usually 25-30% of the total area. Thus it is obvious that the high pressure drop across the draw-off tray causes some improvement of the distribution of the gas.

- 2) In the tangential feed situation, the gas has high velocities at the peripheral area whose width is about the same as the feed inlet diameter (15.2 cm), (see Section 8).

This particular design of draw-off tray is completely blocked on two sides to a width of 23 cm by flat plates, used to collect the liquid, and also blocked by the 15 cm wide sump from the third side, (see Plate 10.1). Thus it can be said that about 70% of the area of the high swirling air velocity is blocked. This blockage improves the distribution of the gas

below the bed.

#### 10.4 Conclusions

From this section of the work it could be concluded that the internal cylinder is doing a little to improve the distribution of the gas and that most of the improvement is caused by the particular design of draw-off tray used in this work.

#### 10.5 Comments on the Test of Existing Distributors

The results summarized on Graph 10.5 not only reveal interesting differences between different methods of gas distribution, but they provide an illustration of the use of the proposed method of testing gas distributors. That is; when compared on a  $\phi$  basis, the best distributor is the one which requires the least packed depth for  $\phi$  to reduce to its minimum value. Thus the distributors represented by lines at the bottom of Graph 10.5 are better than those represented by lines at the top.

It is also significant that when no distributor is used with a tangential feed, a packed depth of 36 cm is not enough to reach  $\phi_{\min}$  and a well distributed gas flow. This is equivalent to a packed depth 2.16m on a full-scale plant, which is greater than that used in many pump around beds.

There is thus a need to develop improved distributors for this duty, and since, as noted above, the pump around column modelled here works at a pressure of 20 mmHg, the distributors used in practice are required to have a negligible pressure drop.

The development of new designs of gas distributors is described in the next section.

PLATE 10.1



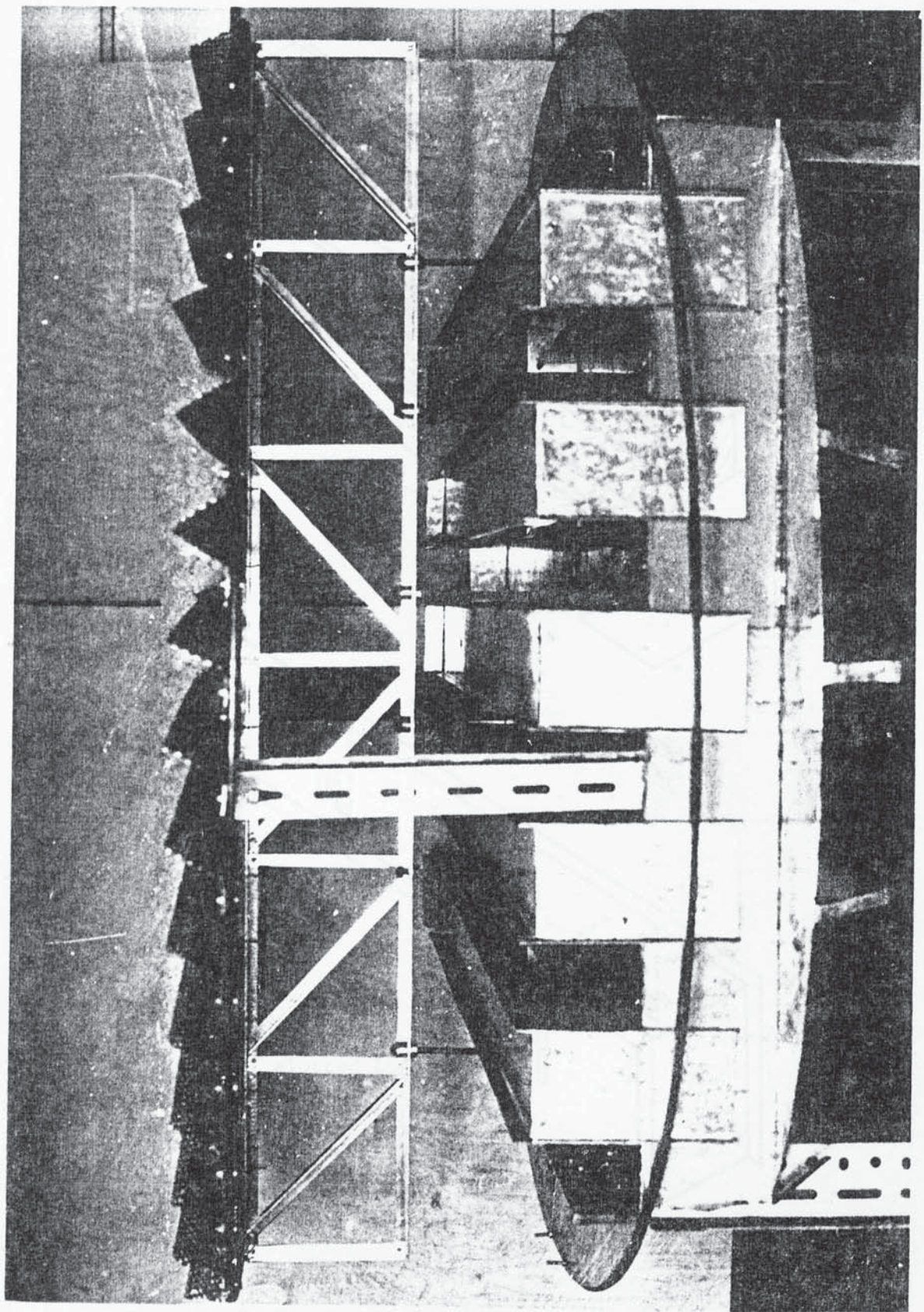




Fig. 10.1   Diagram of slots and  
Troughs in the Draw-off  
Tray

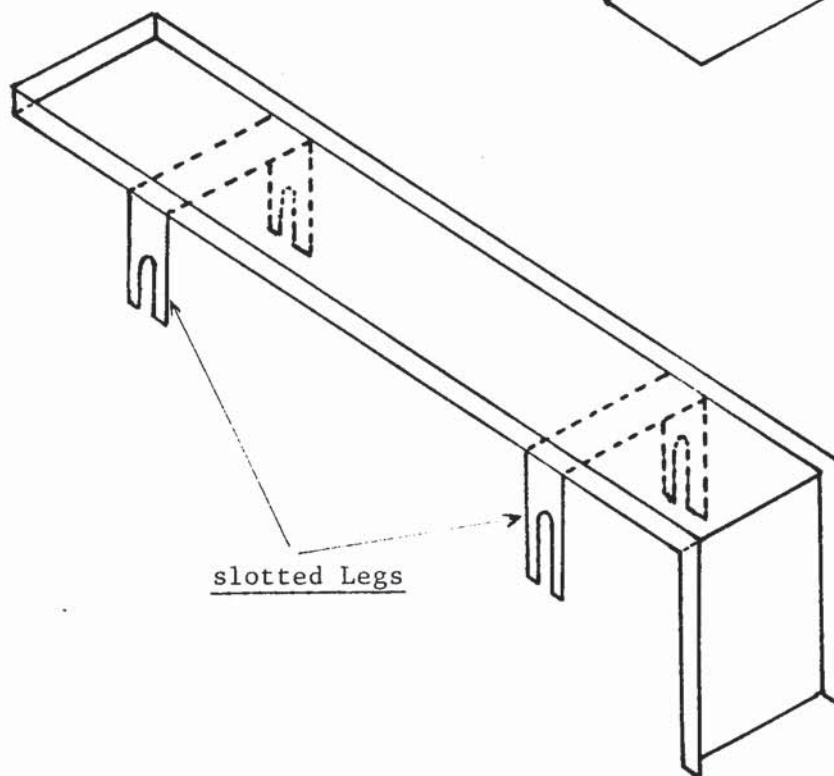
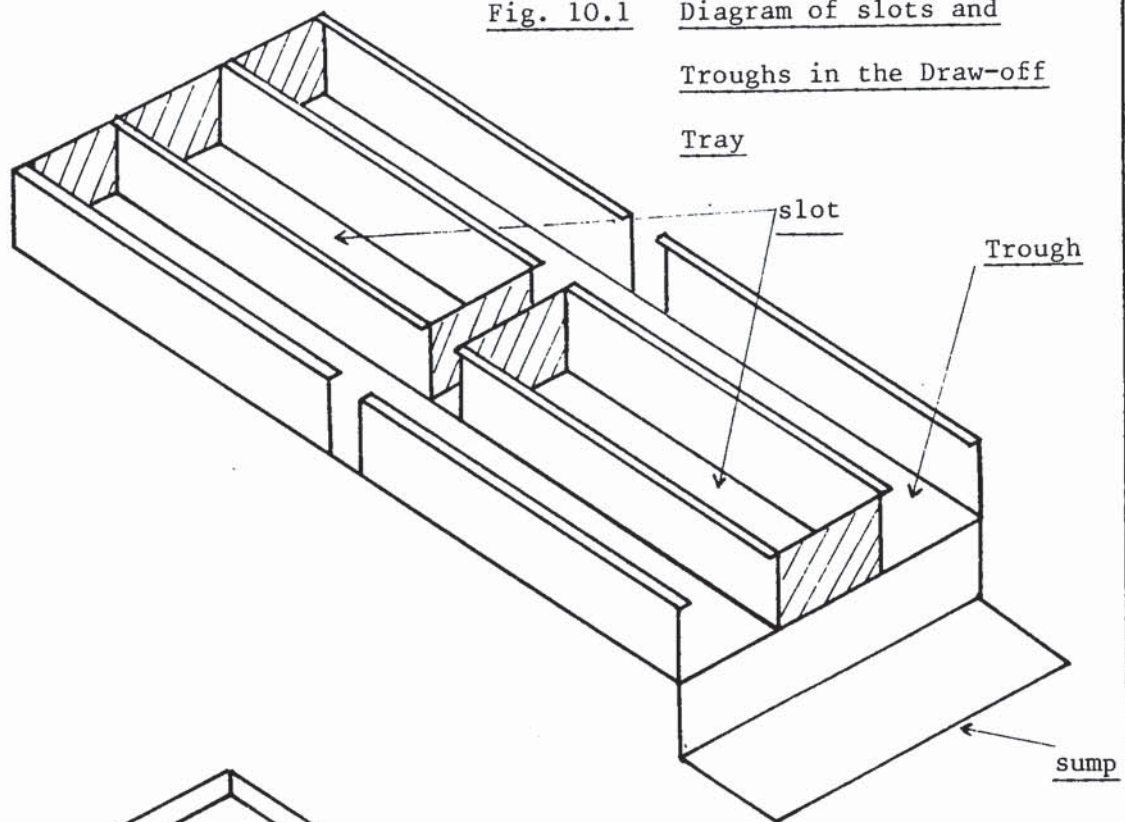


Fig. 10.2   Diagram of the Cap

Fig. 10.3    Cross Section in the Draw-Off Tray

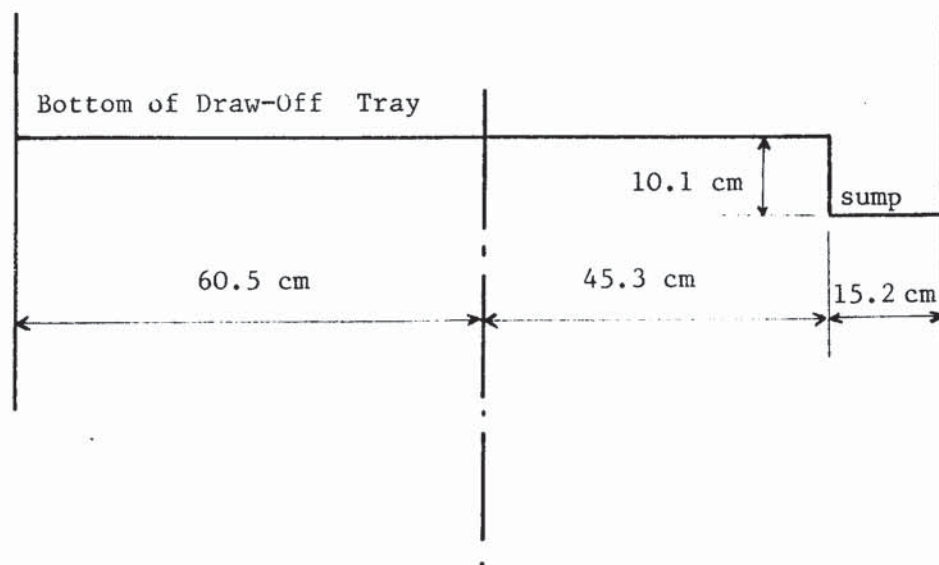
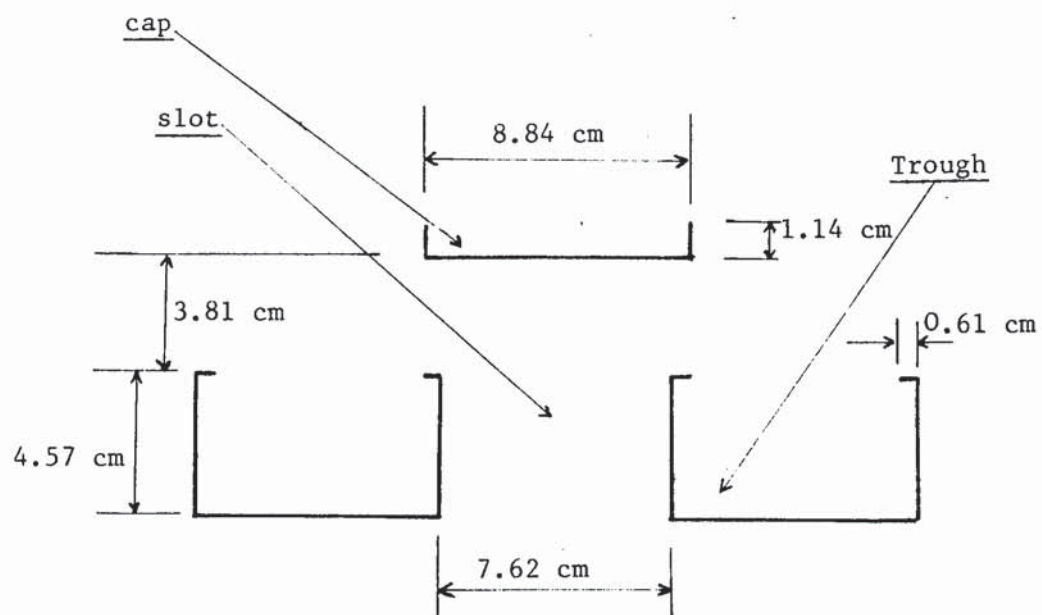


Fig. 10.4    Longitudinal Cross Section in the Draw-off Tray



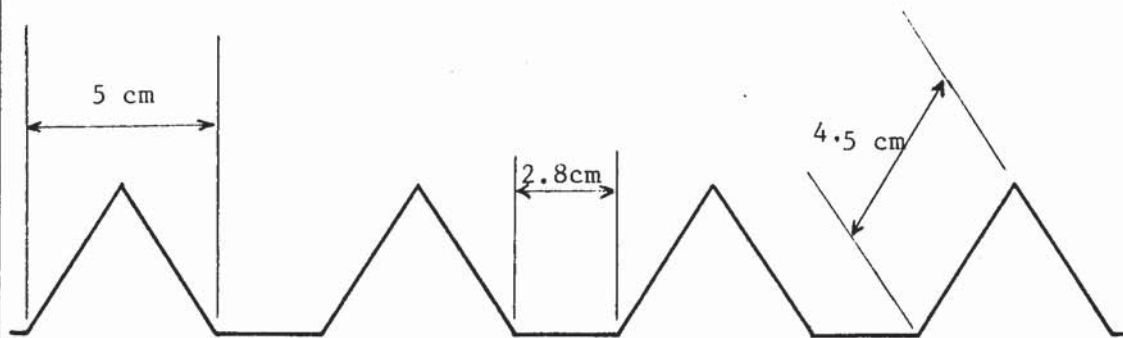


Fig. 10.5 Cross-section in the Support Plate

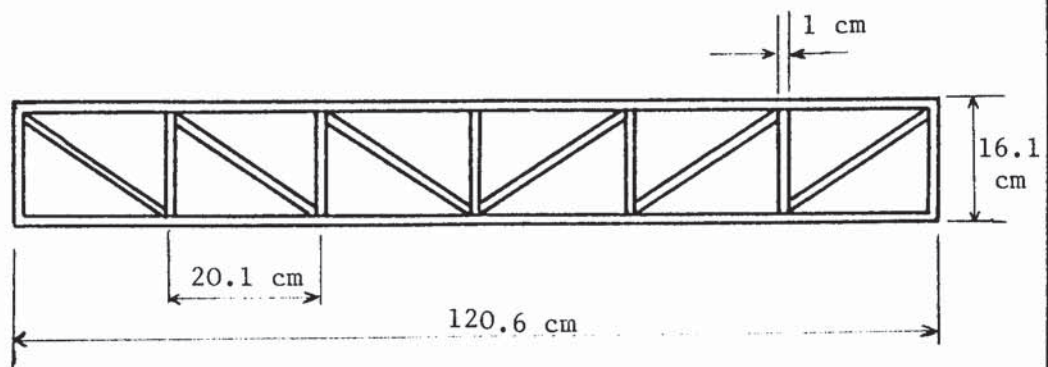


Fig. 10.6 Diagram of the Lattice Support Beam

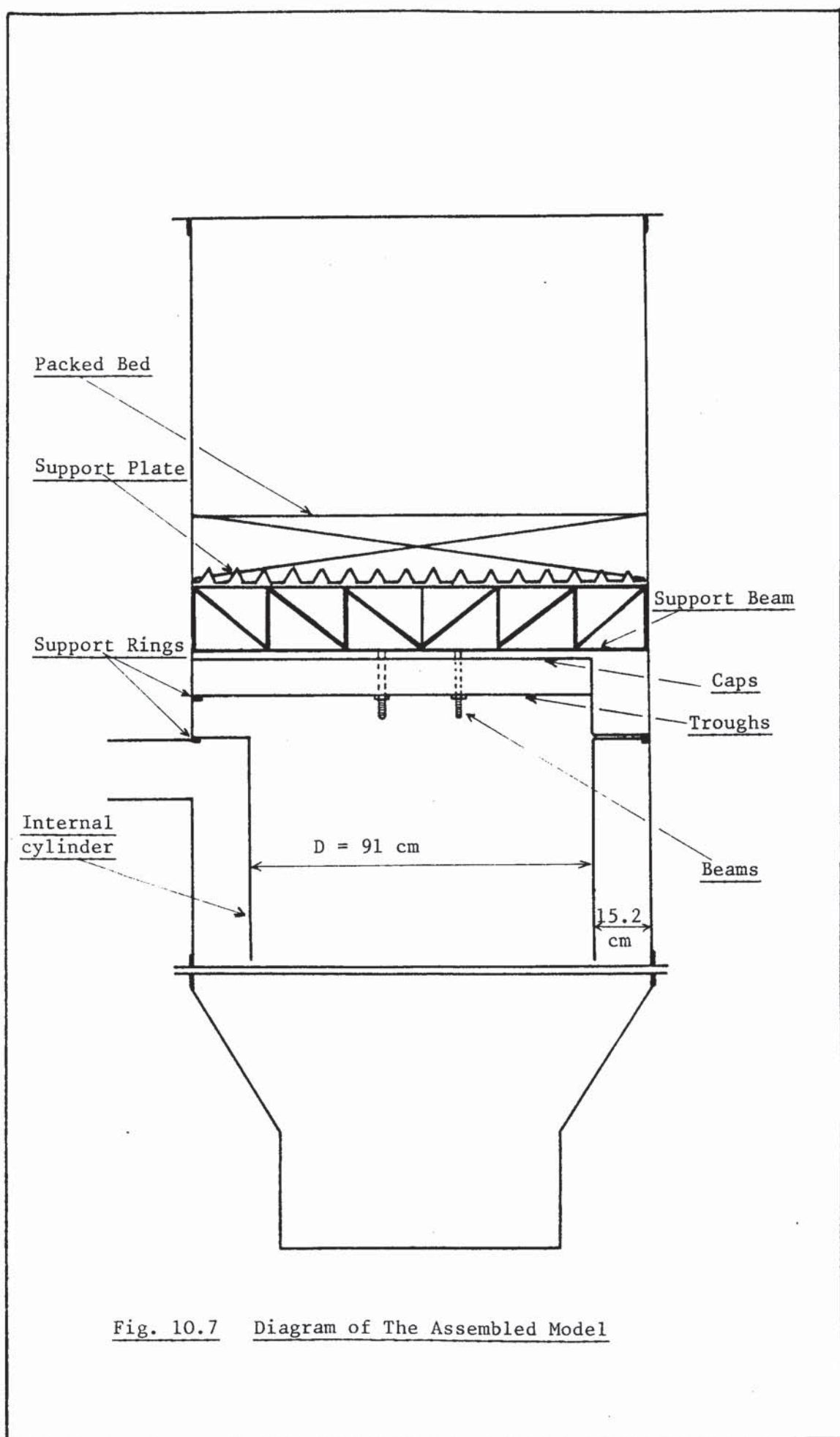


Fig. 10.7 Diagram of The Assembled Model

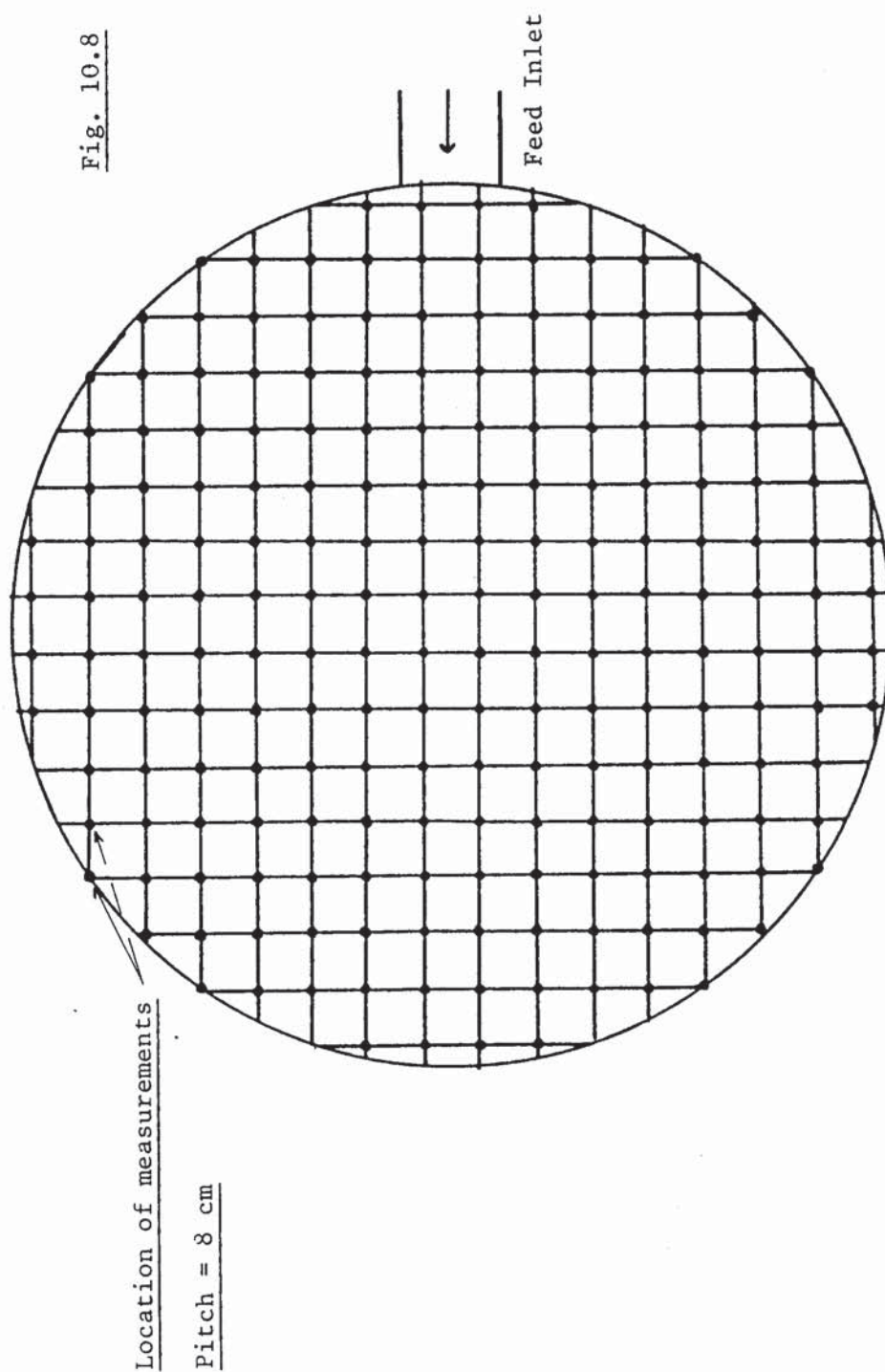


Fig. 10.8 Configuration of 200 Point  
Velocity Measurements taken  
above the 122 cm Diameter  
Bed with Radial Feed Inlet

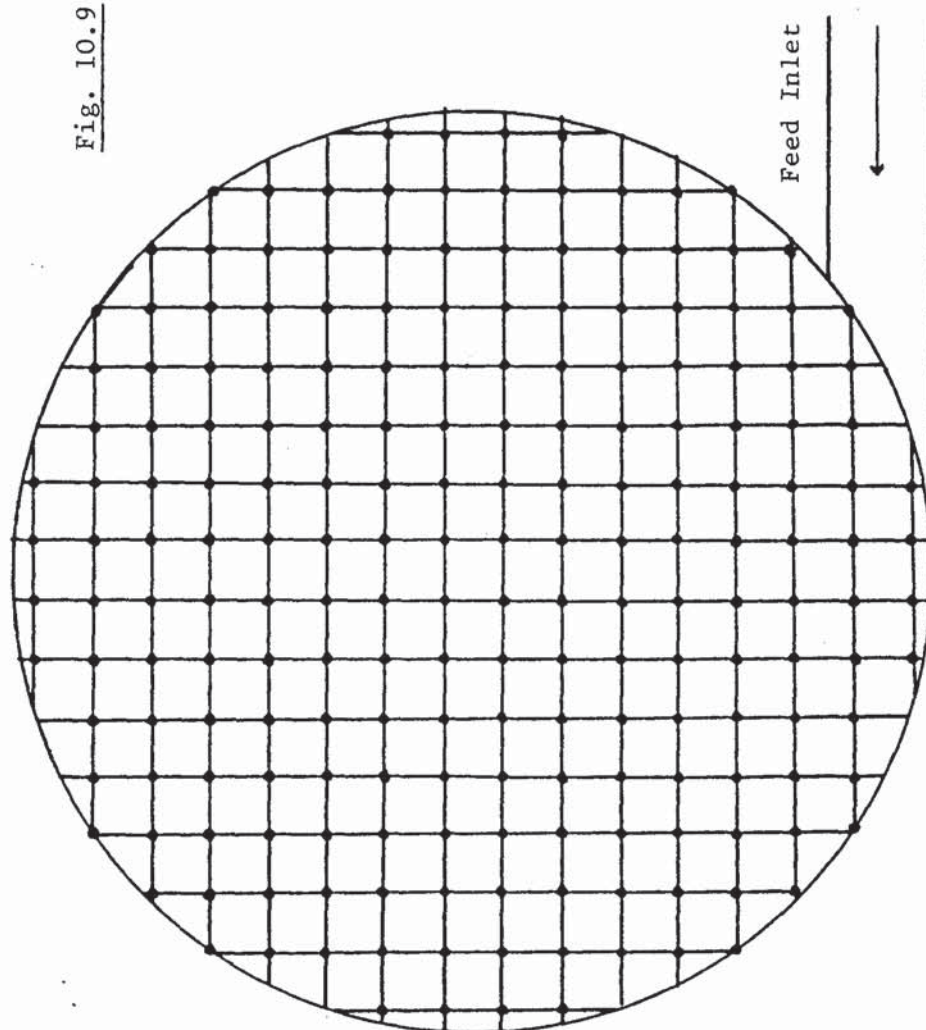


Fig. 10.9

Configuration of 200 point  
Velocity Measurements taken  
above the 122 cm Diameter Bed  
with Tangential Feed Inlet

Table 10.1    Maldistribution Factor,  $\phi$ , for Existing Designs of  
Gas Distributors at Different Packed Depths

Feed Inlet	Gas distributor	P.H. (cm)	Re	$\phi$
Radial	None	0	46,000	836
		6	"	210
		12	"	69
		30	"	21.4
Radial	Draw-off tray	0	46,000	98
		6	"	27
		12	"	22.6
		30	"	19
		36	"	19
		30	80,000	19
Tangential	None	12	46,000	148.5
		18	"	58
		30	"	26
Tangential	Draw-off tray	6	46,000	60
		12	"	38
		24	"	21
Tangential	Internal Cylinder	6	46,000	337.6
		12	46,000	110
		30	46,000	25

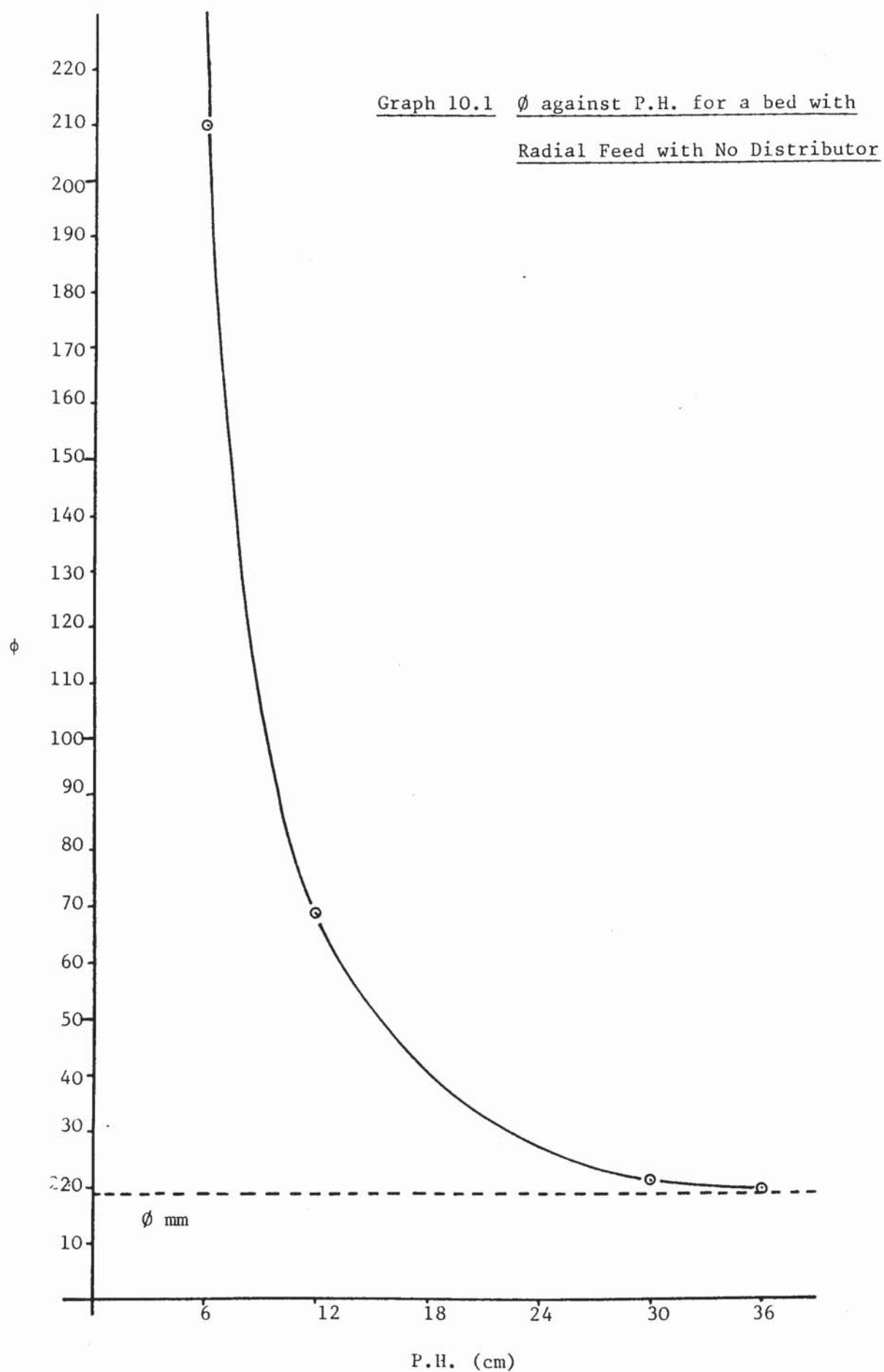




Fig. 10.10    Response surfaces for packed beds with  
radial feed and no distributor

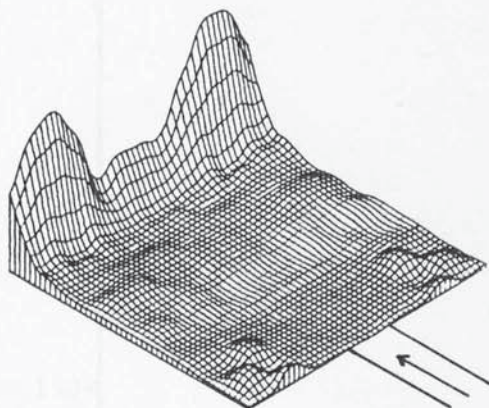


Fig. 10.10a    P.H. = 0

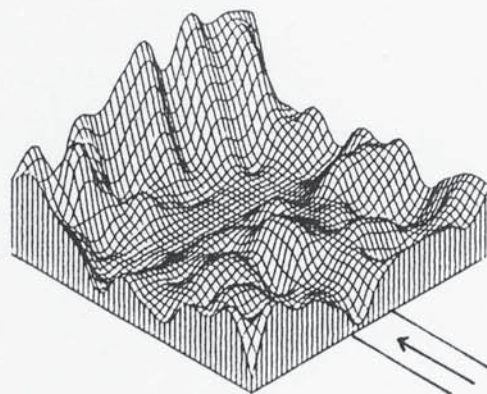


Fig. 10.10b    P.H. = 6 cm

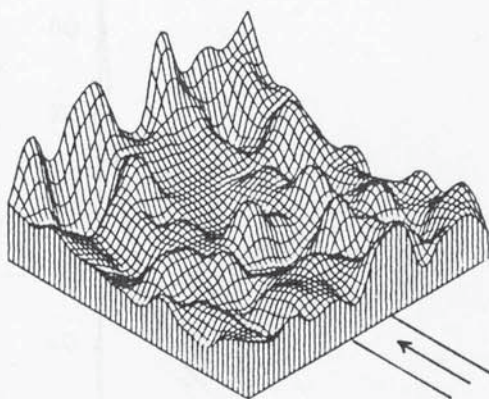


Fig. 10.10c    P.H. = 12cm

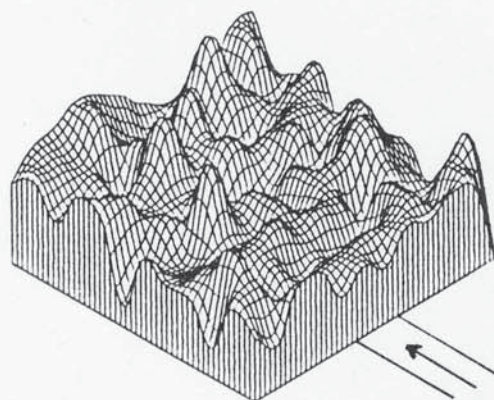


Fig. 10.10d    P.H. = 30 cm

Fig. 10.2  $\phi$  against P.H. for a Bed with Radial  
Feed and Draw-off Tray

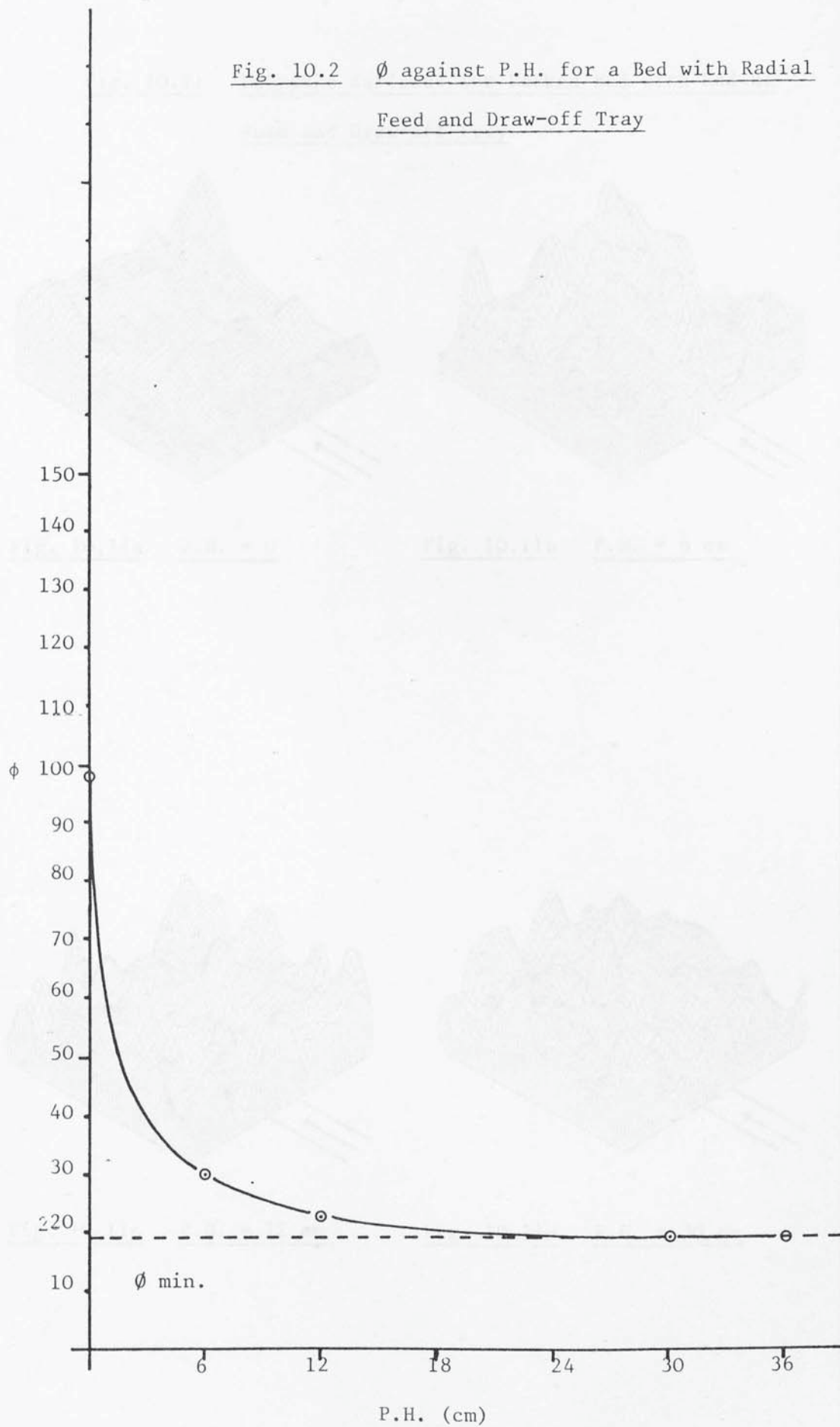




Fig. 10.11    Response Surfaces for Packed Bed with Radial  
Feed and Draw-off Tray

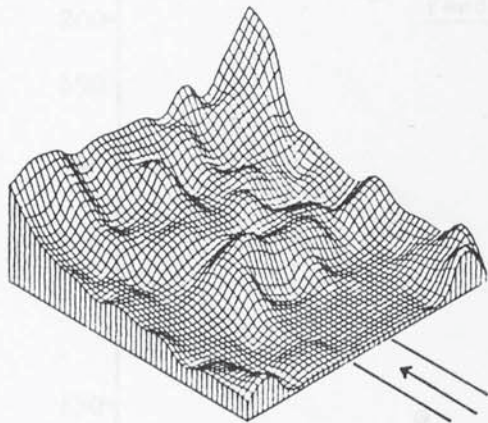


Fig. 10.11a    P.H. = 0

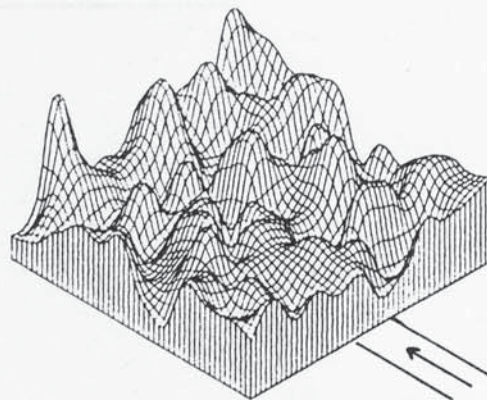


Fig. 10.11b    P.H. = 6 cm

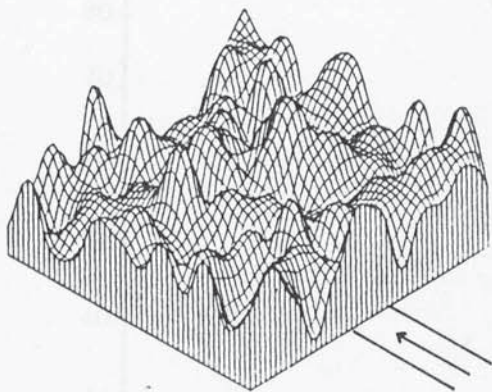


Fig. 10.11c    P.H. = 12 cm

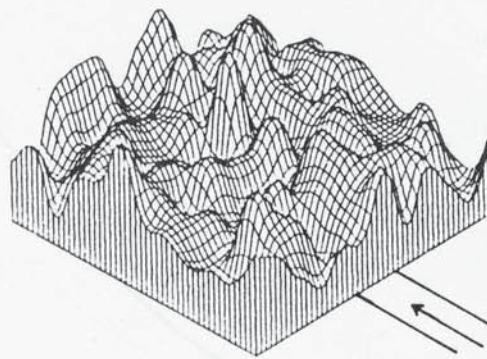


Fig. 10.11d    P.H. = 30 cm

Graph 10.3     $\phi$  against P.H. for a Bed with Tangential  
Feed and No Distributor

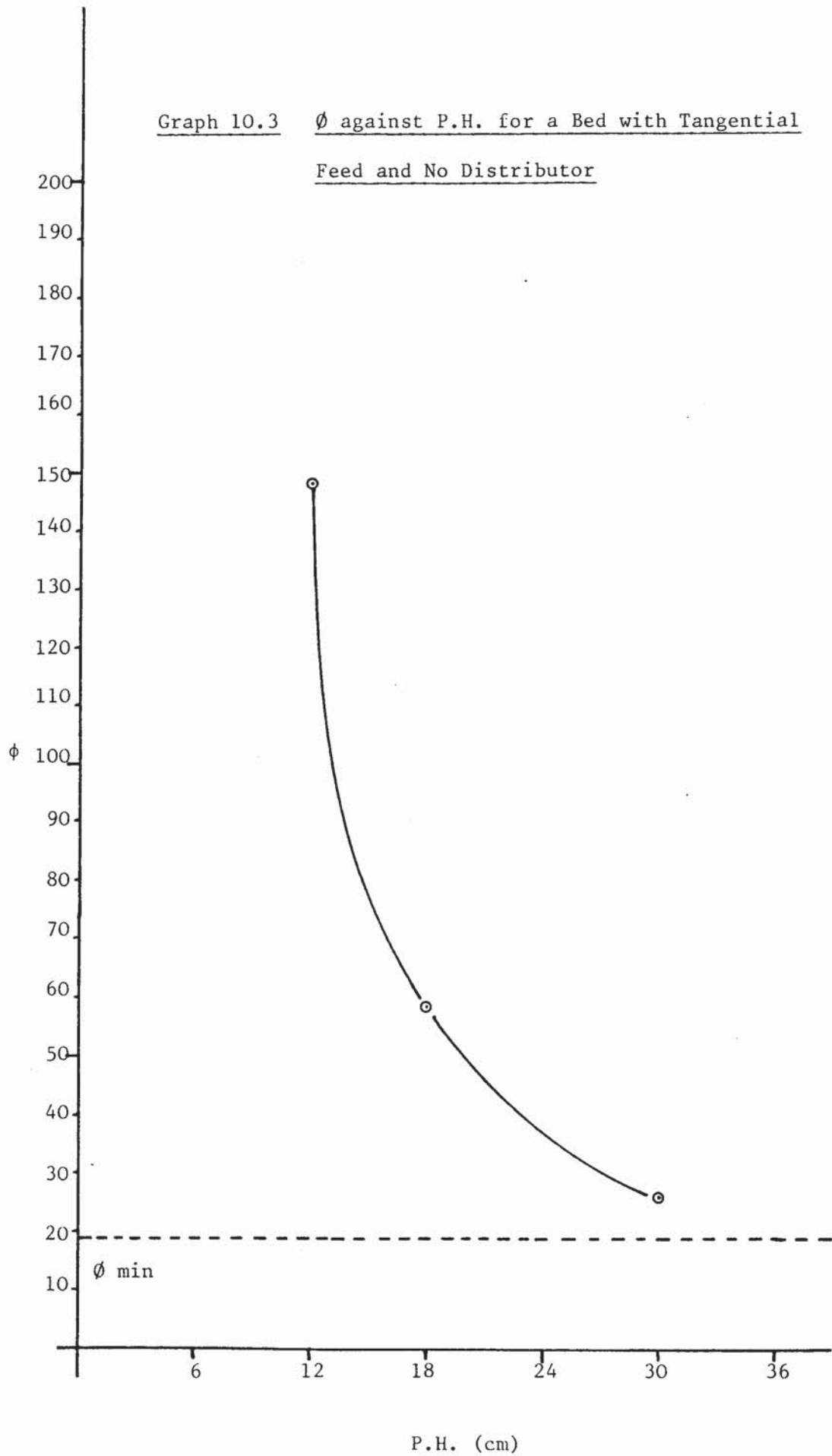


Fig. 10.12      Response Surfaces for Packed Beds with  
Tangential Feed and No Distributor

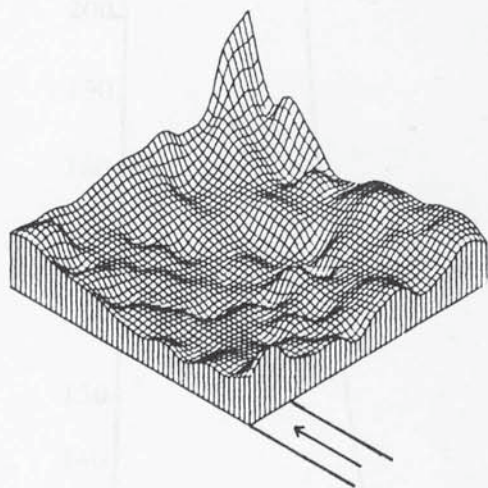


Fig. 10.12a    P.H. = 6 cm

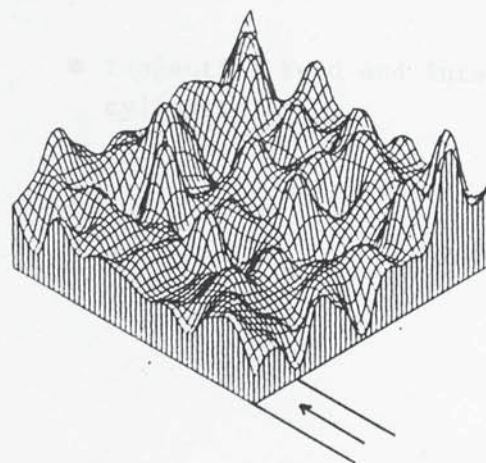


Fig. 10.12b    P.H. = 30 cm

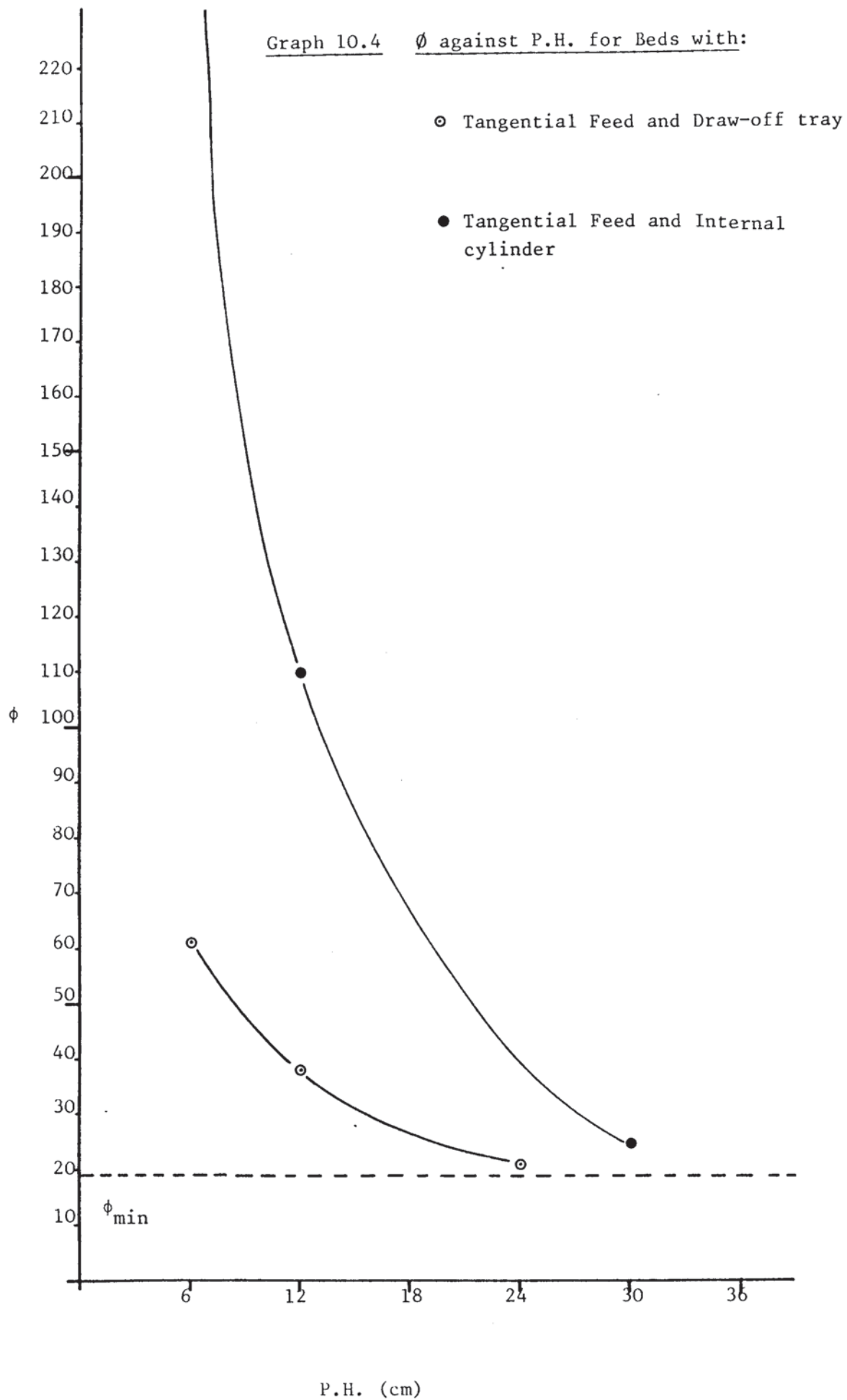




Fig. 10.13 Response Surfaces of Packed Beds with Tangential  
Feed Inlet and Draw-off Tray

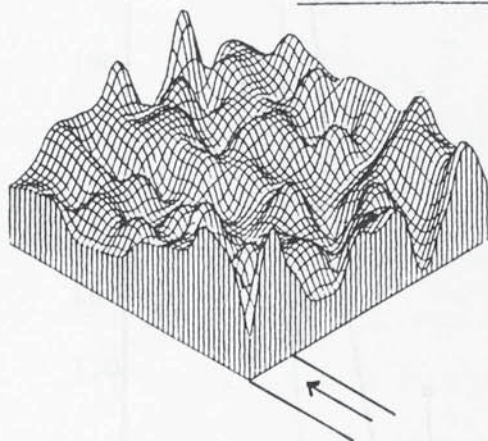


Fig. 10.13a P.H. = 6 cm

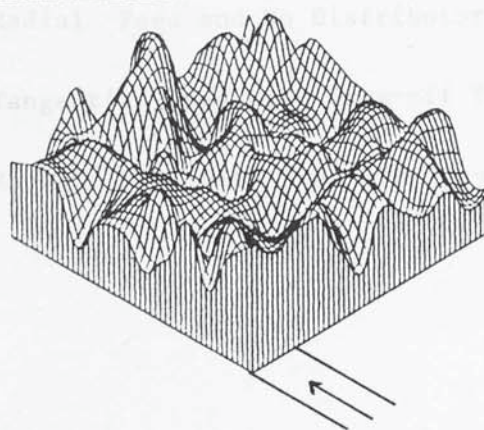


Fig. 10.13b P.H. = 12 cm

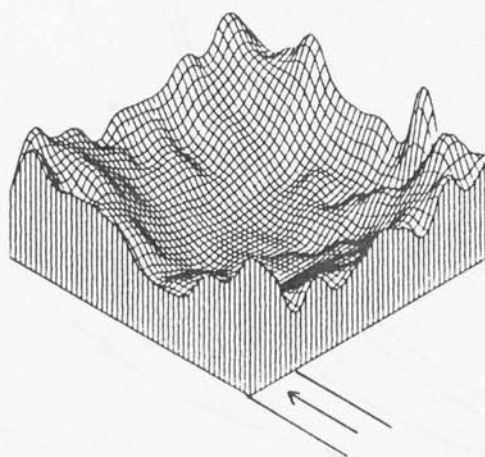
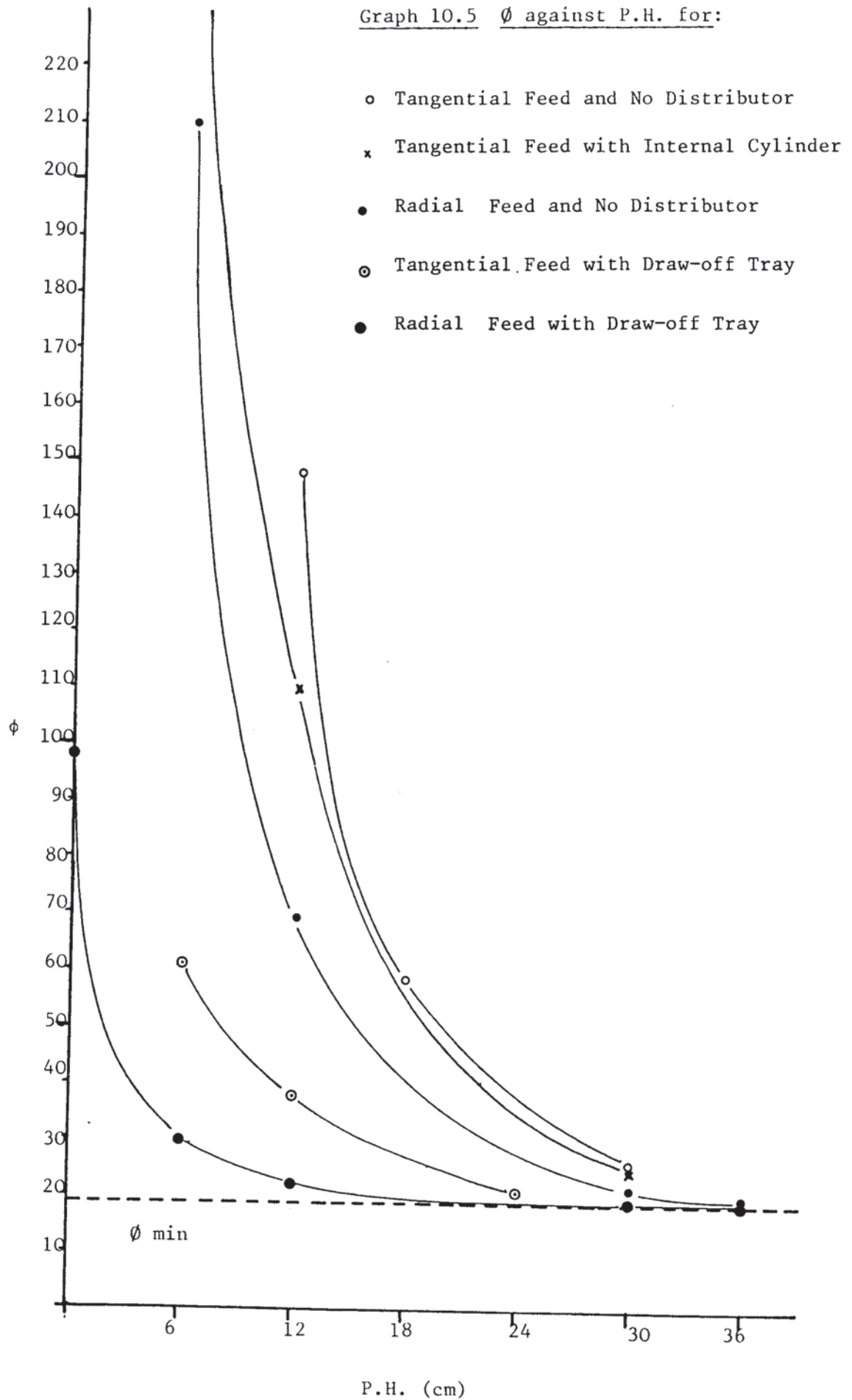


Fig. 10.14 Response Surface for a 6 cm deep Packed Bed with  
Tangential Feed Inlet and Internal Cylinder

Graph 10.5  $\phi$  against P.H. for:



SECTION XI  
DEVELOPING AND TESTING NEW DESIGNS OF  
GAS DISTRIBUTORS

Introduction

It was found, in the last section, that the particular design of the draw-off tray which Norton had supplied, is an effective means of improving gas distribution. But there will be situations where it is desired to achieve better gas distribution without fitting a liquid draw-off tray.

This stage of the work is concerned with developing and testing new low pressure drop gas distributors suitable for use in large diameter packed towers.

All the new designs of distributors below are modifications to the known use of tangential feed, combined with internal cylinder within the column, described in Section 9.1.3.

11.1 Modifications to the Internal Cylinder Gas Distributor

The gas emerging from the annulus is well distributed around the bottom of the internal cylinder, but is still swirling. It was observed, that at short packed depths associated with a swirling gas below the bed, a downward flow of air results in the middle of the bed.

All modifications are designed to remove this swirl.

11.1.1 Modification (A): Internal Cylinder with Four Slots and Cross Baffles near the Bottom of the Cylinder

This is shown separately in Fig. 11.1a and together with the column in Fig. 11.1b. Four cuts of 15 cm deep and 45 cm long

were made equally spaced around the lower end of the circumference of the internal cylinder, and the metal between the cuts was bent towards the middle of the cylinder. The four baffles formed were attached to each other at the centre to form a cross baffle. It is intended by this design to cause the swirling air at the bottom of the internal cylinder to change direction and go upwards vertically.

11.1.2 Modification (B): Internal Cylinder with Vertical Baffles in the Annulus

This distributor is shown in Fig. 11.2. Six vertical radial plates of depth of 20.3 cm and width of 15.25 cm were attached at the lower part of the annular space between the internal cylinder and the column wall. The number then increased, as shown in Fig. 11.3 and Plate 11.1, to 24 equally spaced plates to form air passages of length nearly twice that of the width. These plates are intended to straighten the swirling air at the bottom of the annulus and cause it to go down vertically, i.e. to produce a uniform distribution of gas around the bottom of the internal cylinder, but at the same time remove any swirl of the gas before it reaches the bottom of the internal cylinder.

11.1.3 Modification (C): Internal Cylinder with Internal Baffles

This is shown in Plate 11.2 and Fig. 11.4. The feed is again distributed equally around the bottom edge of the internal cylinder, and enters the cylinder with a swirling movement which is removed by two crossed baffle plates 90 cm in length and 45 cm in depth, put towards the top of the cylinder.

## 11.2 Tests on Distributor Modifications A, B and C

Typical unprocessed results are shown in Appendix A5 for Modification A, B with six baffles, B with 24 baffles, C, and Modification C combined with the draw-off tray described in Section 9.1.1. Response surfaces of gas distribution are given in Figs. 10.5-10.9 for the modifications respectively. The maldistribution factor,  $\phi$ , values were obtained and listed in Table 11.1. The modified distributors are compared with the original simple internal cylinder and with each other in Graph 11.1, which shows plots of  $\phi$  values against packed depth. That is; the new modifications are tested by the method proposed in this work, (see Section 6).

## 11.3 Discussion of the Results

All modifications; A, B and C, provide a significant improvement in the quality of gas distribution. The best modified distributor would be that one which produces a distribution of gas, expressed by  $\phi$ , nearest to that described by  $\phi_{\min}$  at the shortest packed depth. That is, the comparison takes place at a packed depth of 6 cm which is equivalent to 36 cm in a real life packed bed.

Therefore, the unmodified internal cylinder is producing maldistribution factor,  $\phi$ , as big as 18-fold the value of  $\phi_{\min}$  at packed depth of 6 cm. Modifications A, B (with 6 baffles), B (with 24 baffles) and C, are producing gas distribution which can be described in terms of  $\phi_{\min}$  as;  $2.25 \phi_{\min}$ ,  $1.76 \phi_{\min}$ ,  $1.58 \phi_{\min}$  and  $1.22 \phi_{\min}$  respectively, (see Table 10.1 and Graph 10.1). That means that Modification C; Internal Cylinder with Internal Baffles, is the best modified gas distributor.



In real life large towers, the performance, capital cost and pressure drop, are the most important guidances in the choice of gas distributor. All modifications mentioned above can be considered as low pressure drop distributors. Then the choice between these modified distributors would be solely related to their performances and capital costs, the best gas distributor would be that of best performance with least capital cost.

Modification A: Internal Cylinder with Four Slots and Cross Baffles, would be the cheapest of all, since the baffles used originate from the body of the internal cylinder itself.

However, Modification B: Internal Cylinder with Six Annular Baffles, is of better performance as well as providing an extra mechanical strength to the internal cylinder by attaching it at the bottom to the column wall.

Modification C: Internal Cylinder with Internal Baffles is providing best gas distribution;  $1.22 \phi_{\min}$  within 36 cm in the real life packed bed. But it would be very expensive to build large internal baffles; two crossed baffles of dimensions of 540 cm x 270 cm in the real life distributor. But those baffles might replace the expensively constructed lattice support beam whose small model is shown in Fig. 10.6 in supporting the tower internals.

#### 11.4 Conclusions

From above it could be concluded that Modification C: Internal Cylinder with Internal Baffles is the best low pressure gas distributor and it is believed that it would be suitable for large diameter towers with or without a liquid draw-off tray.



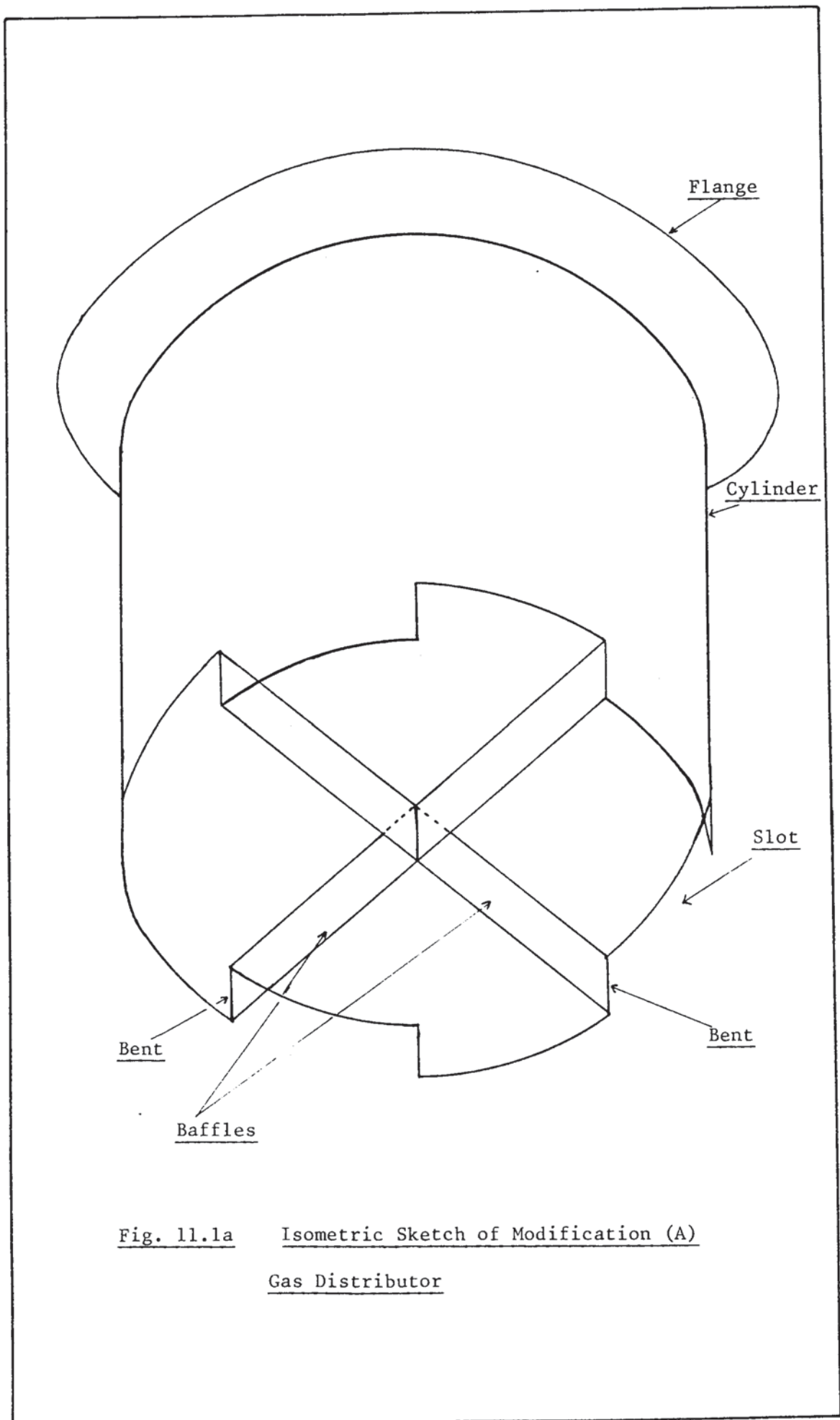


Fig. 11.1a      Isometric Sketch of Modification (A)  
Gas Distributor

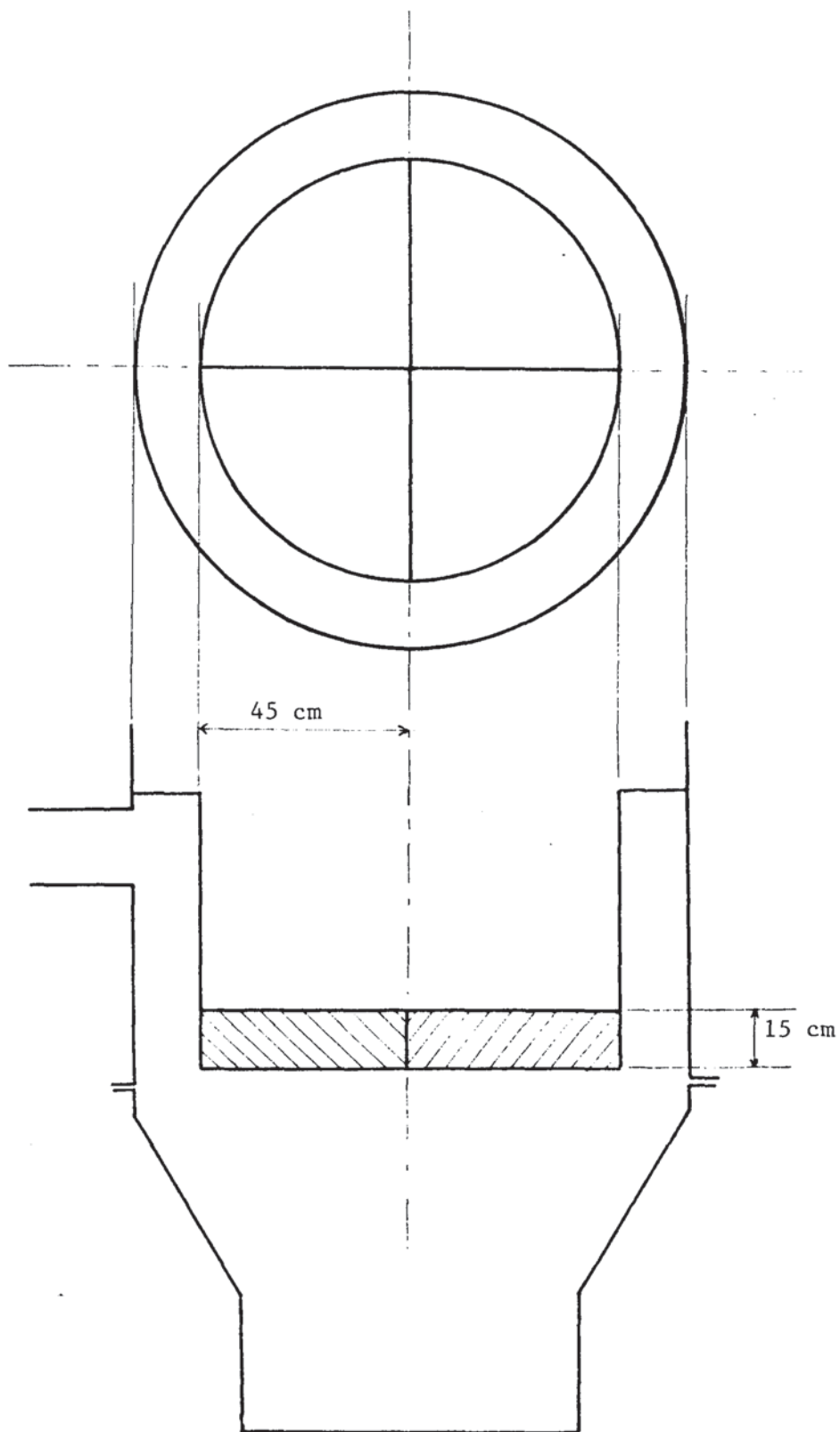
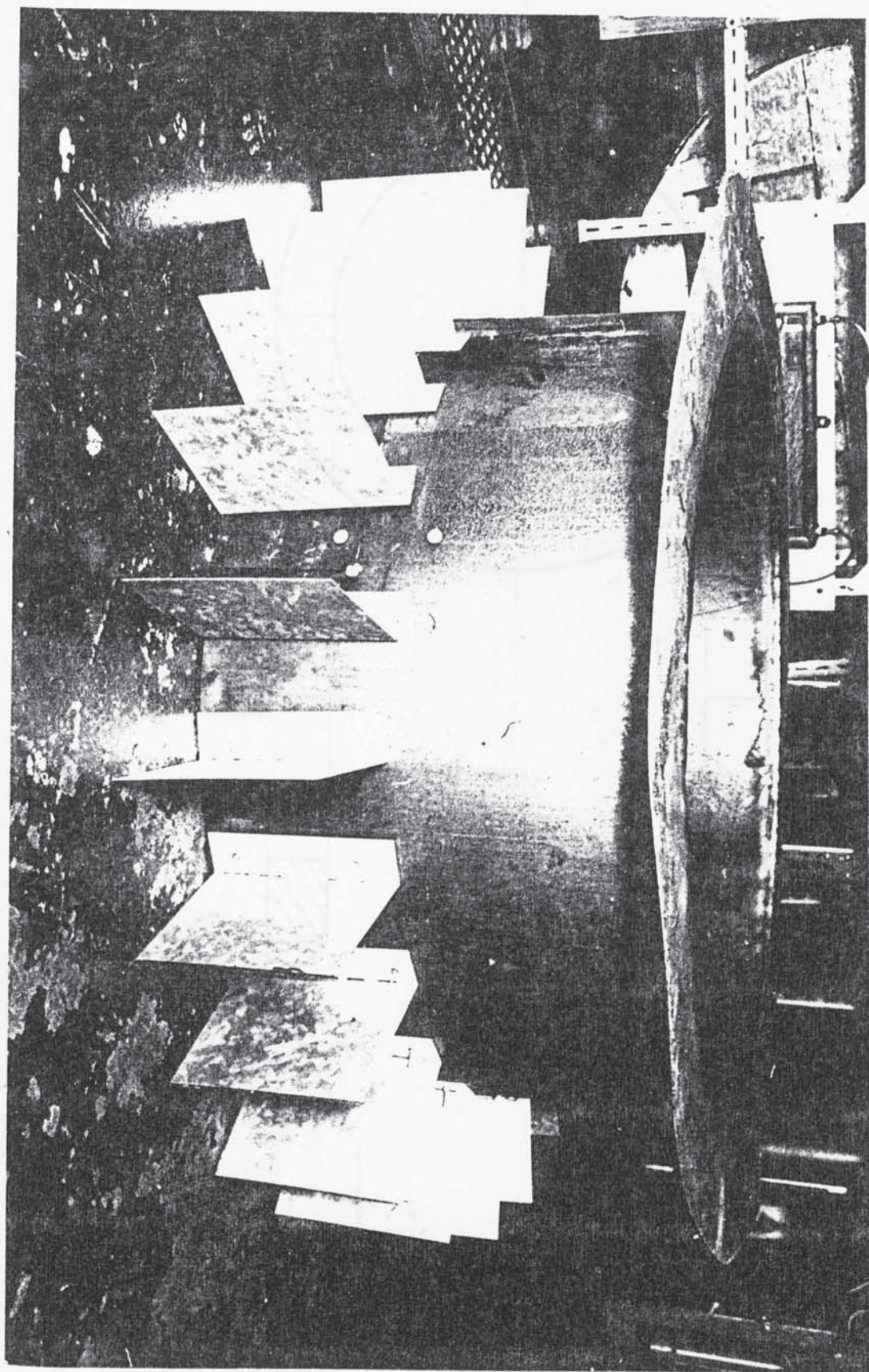


Fig. 11.1b    Modification (A) within the Column

PLATE 11.1





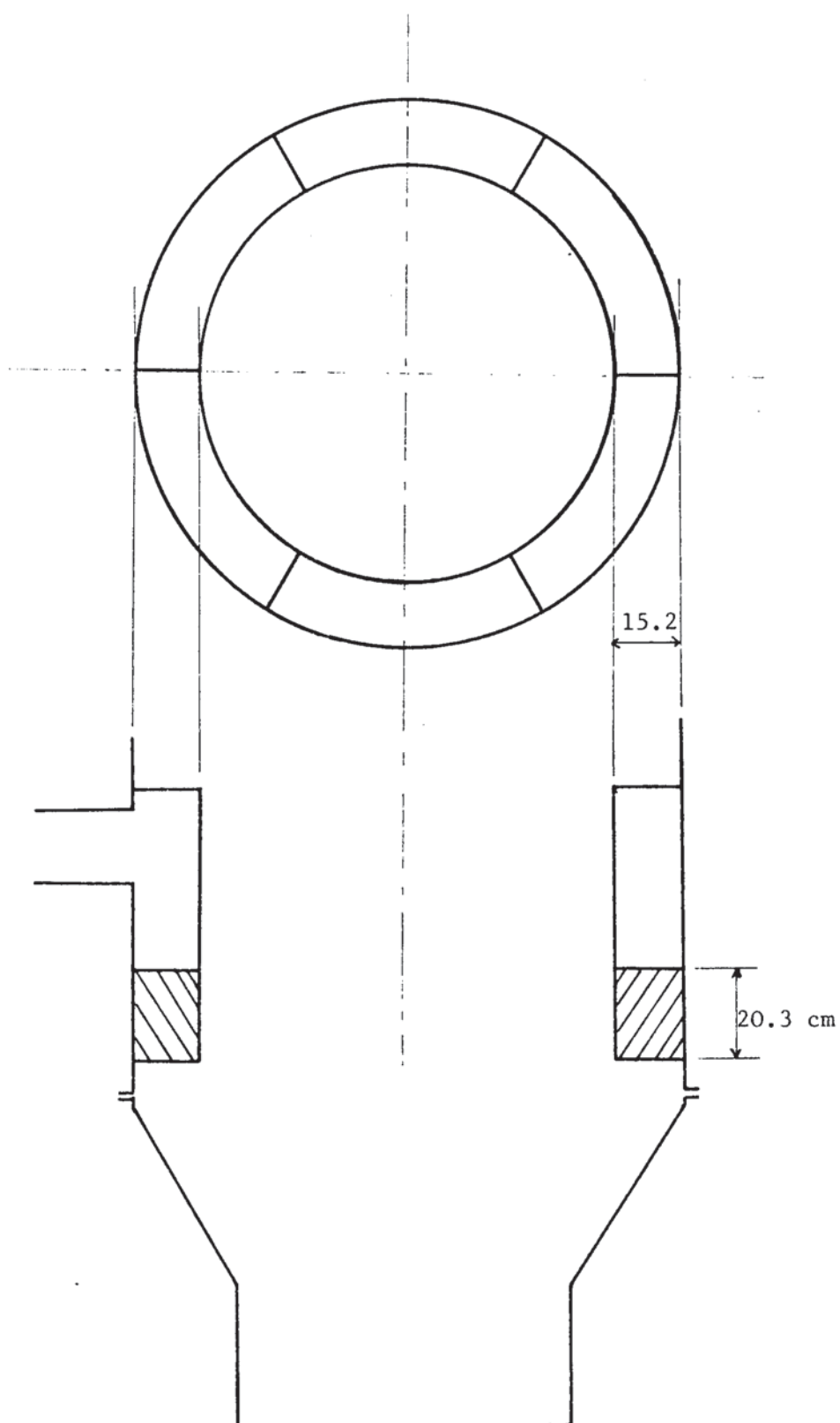


Fig. 11.2    Modification (B) with six annular Baffles

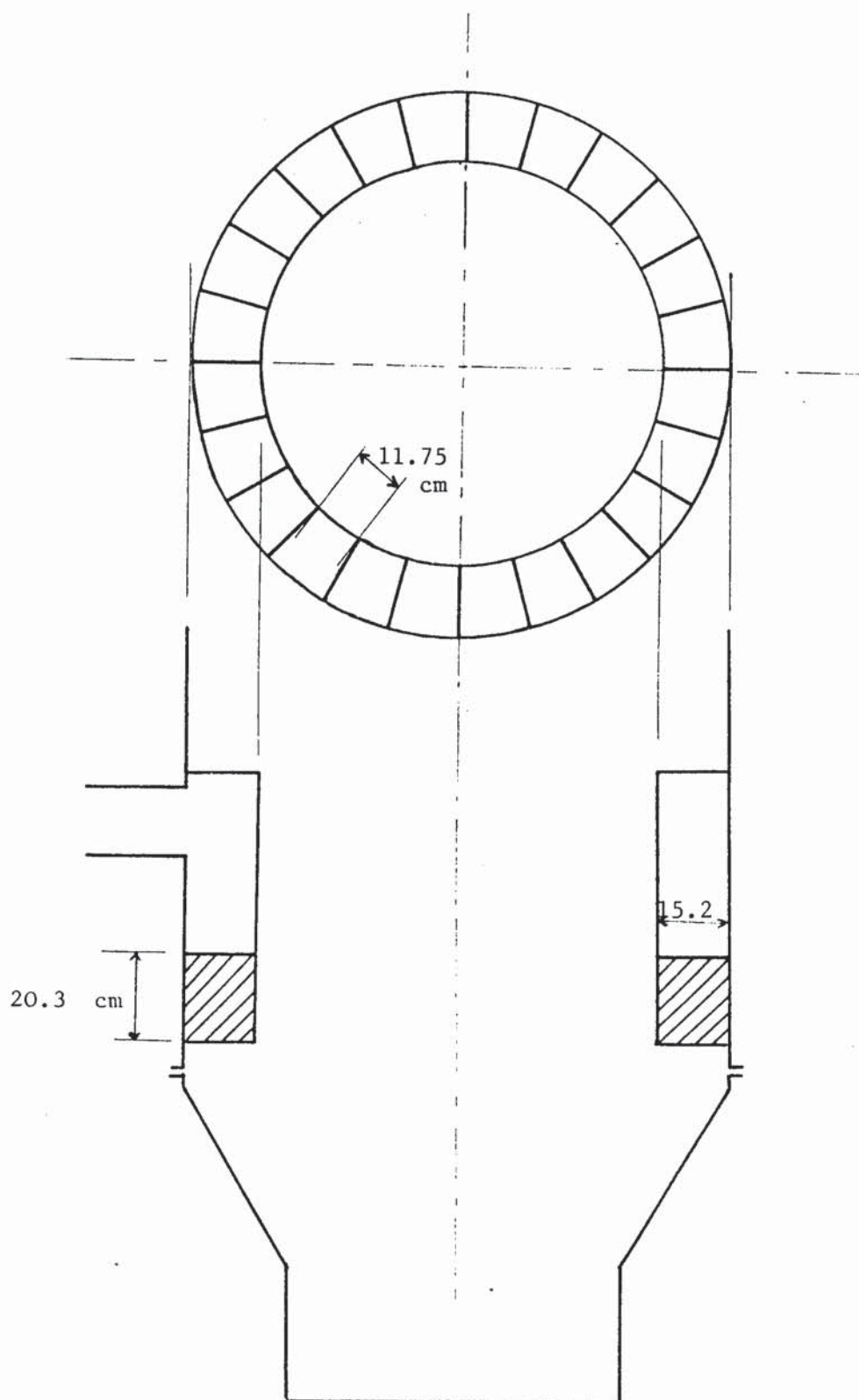
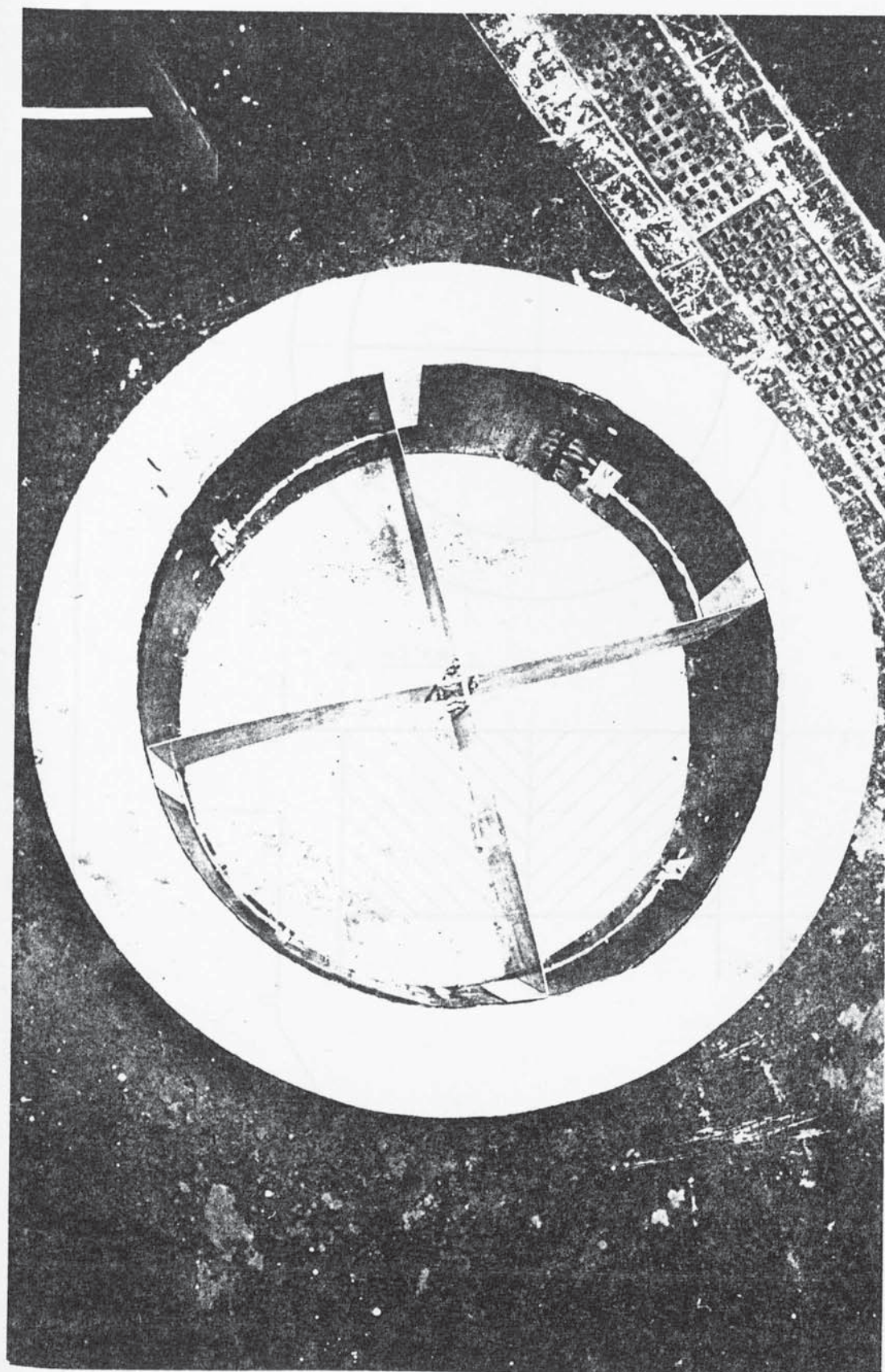


Fig. 11.3    Modification B with 24 Annular Baffles



PLATE 11.2



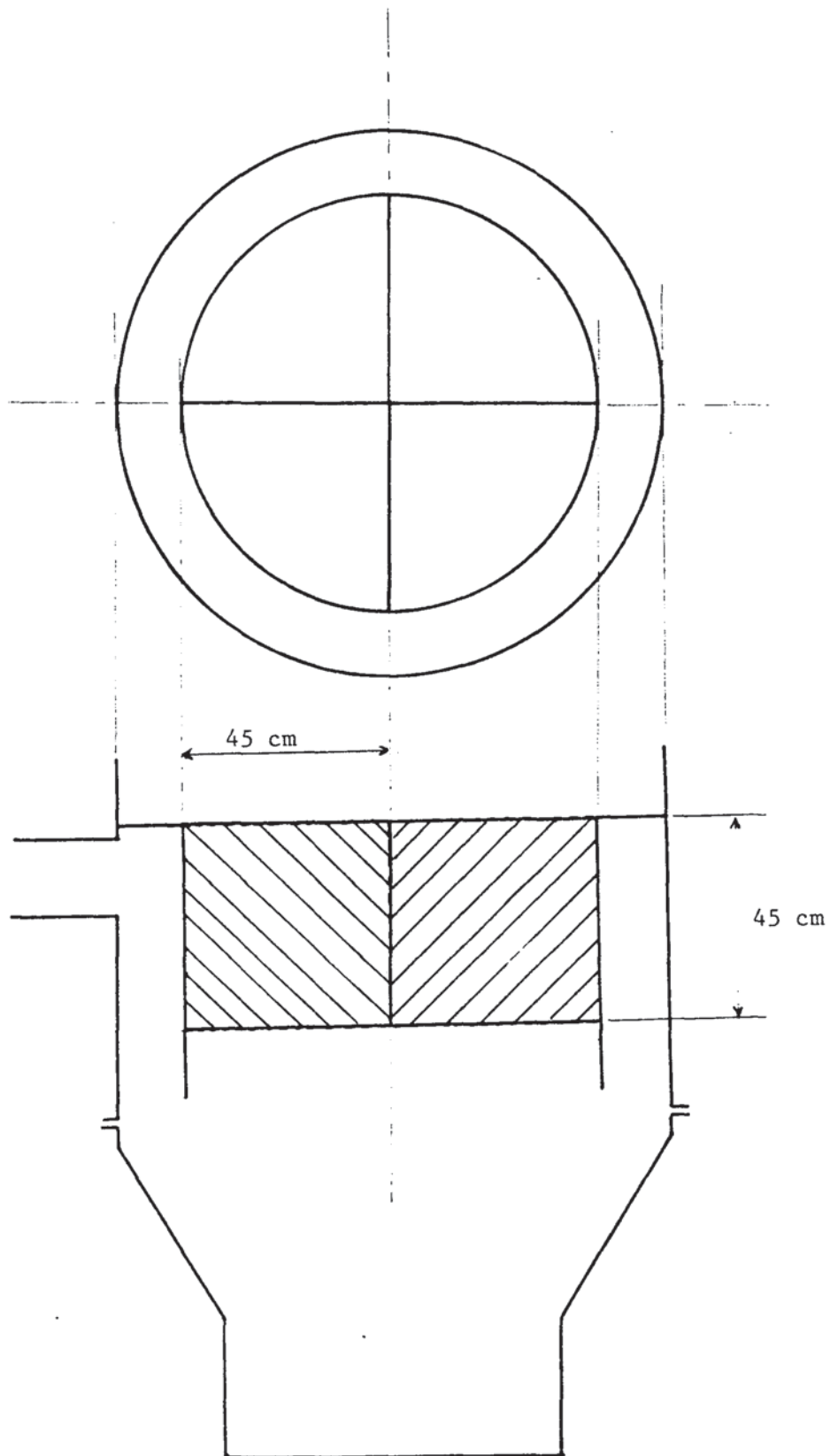


Fig. 11.4    Modification C

Table 11.1    Maldistribution Factor,  $\phi$ , for the Modified  
Gas Distributors at Different Packed  
Depths

Distribution	P.H. (cm)	$\phi$
Modification A	6	42.7
	12	31.3
	30	19
Modification B: with six plates  with 24 plates	6	33.5
	12	26
	30	19
	6	30
	12	24.8
	30	19
Modification C: Alone  plus draw-off tray	6	23.3
	12	21.1
	30	19
	6	22.8
	12	21
	30	19

Graph 11.1 Variation of  $\phi$  with P.H. for:

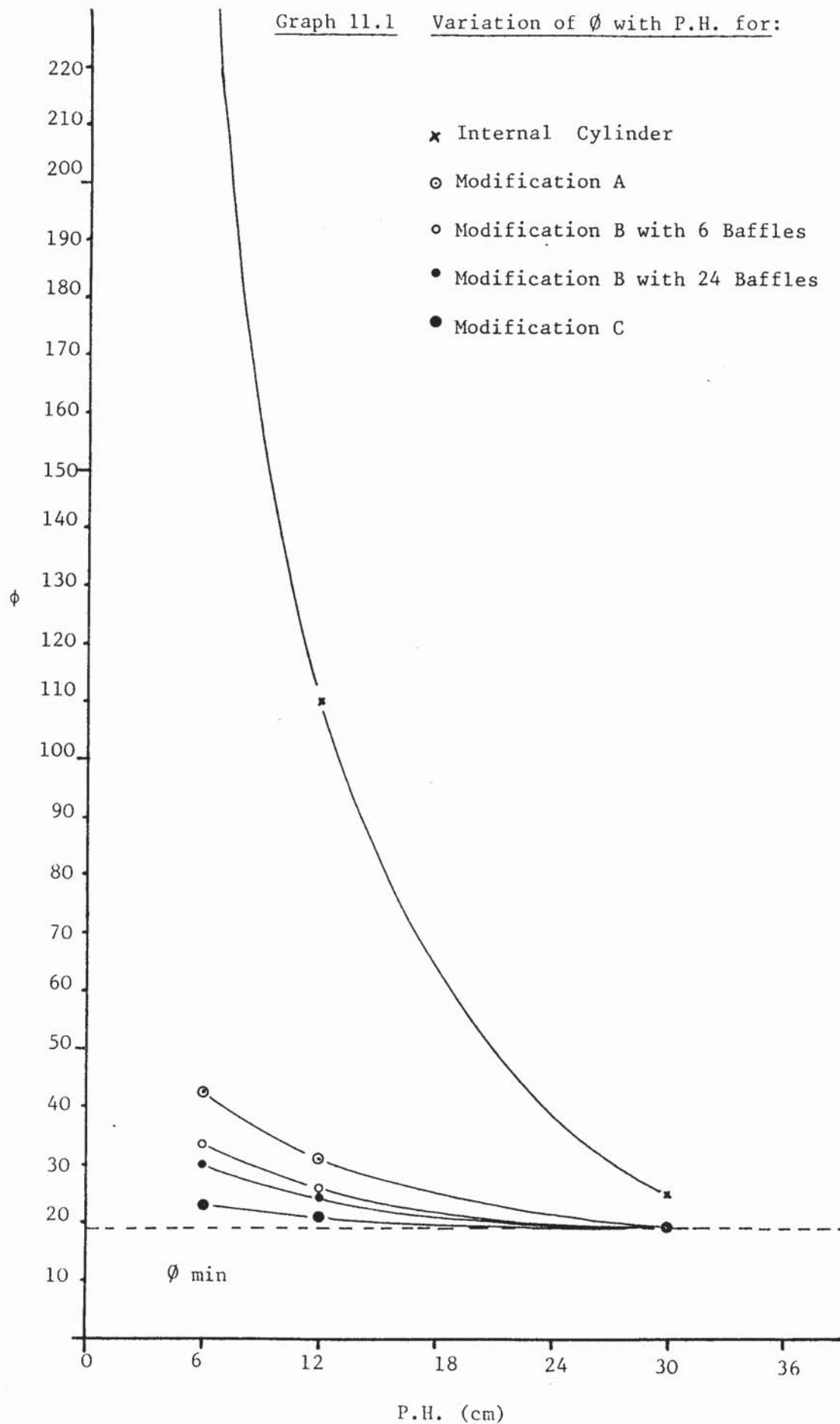




Fig. 11.5   Modification A

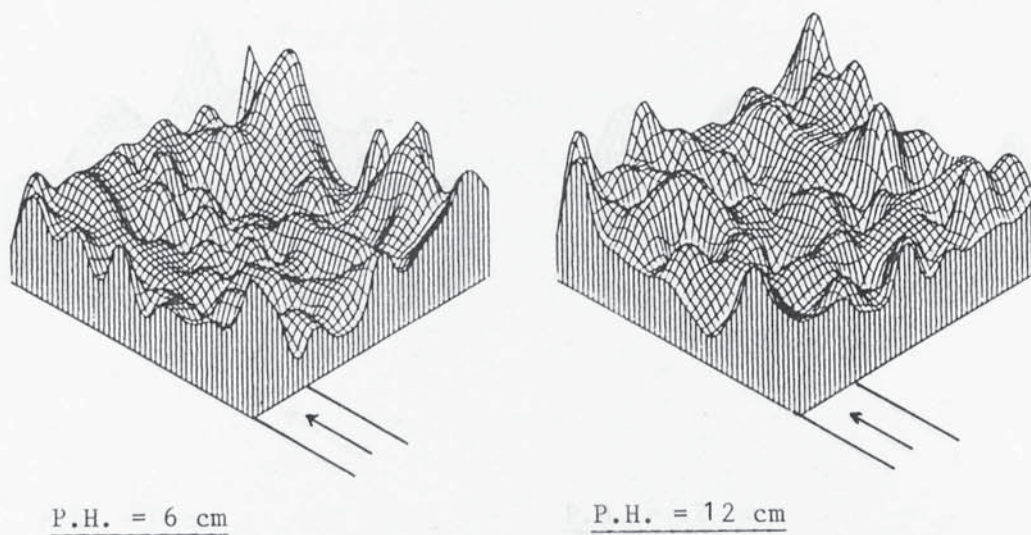


Fig. 11.6   Modification B with six baffles

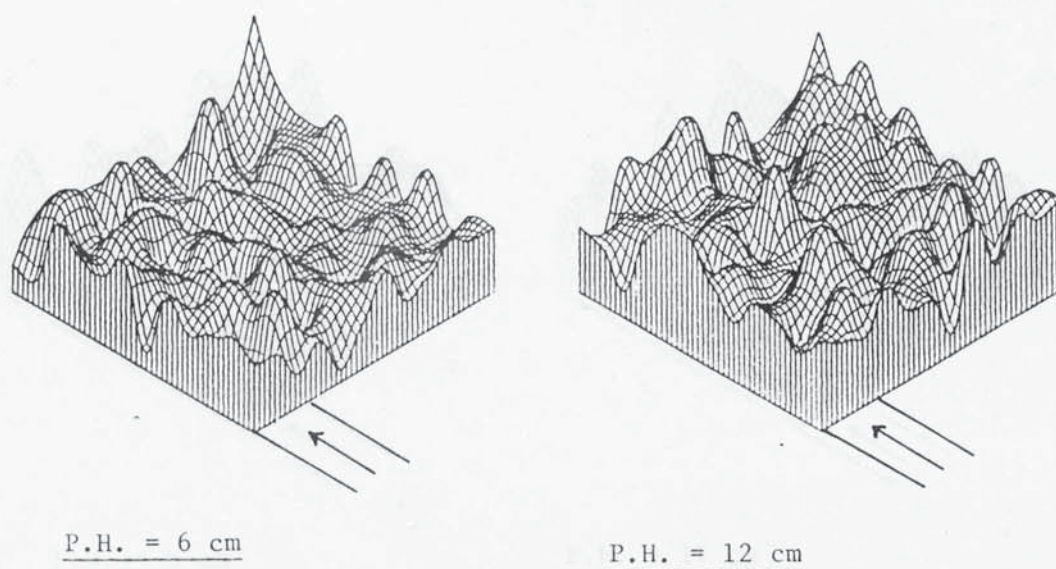
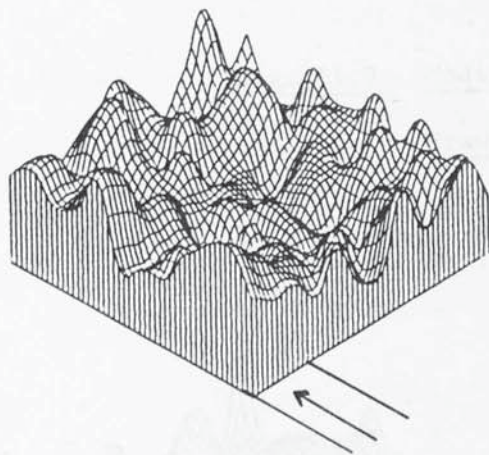
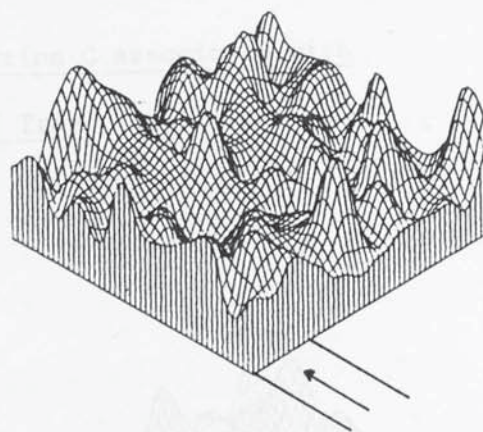




Fig. 11.7    Modification B with 24 Baffles

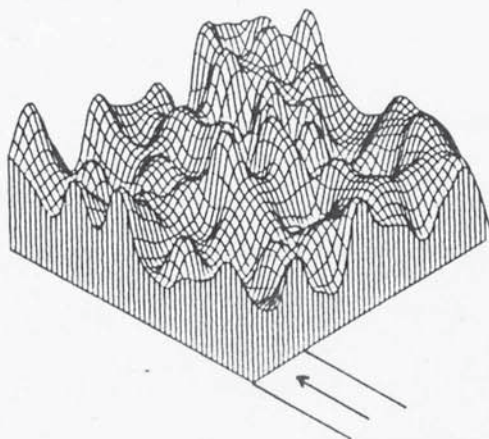


P.H. = 6 cm

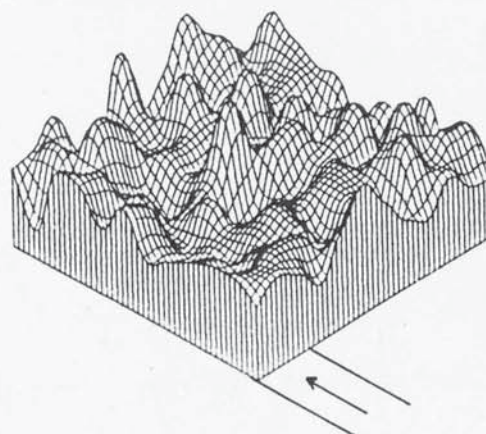


P.H. = 12 cm

Fig. 11.8    Modification C

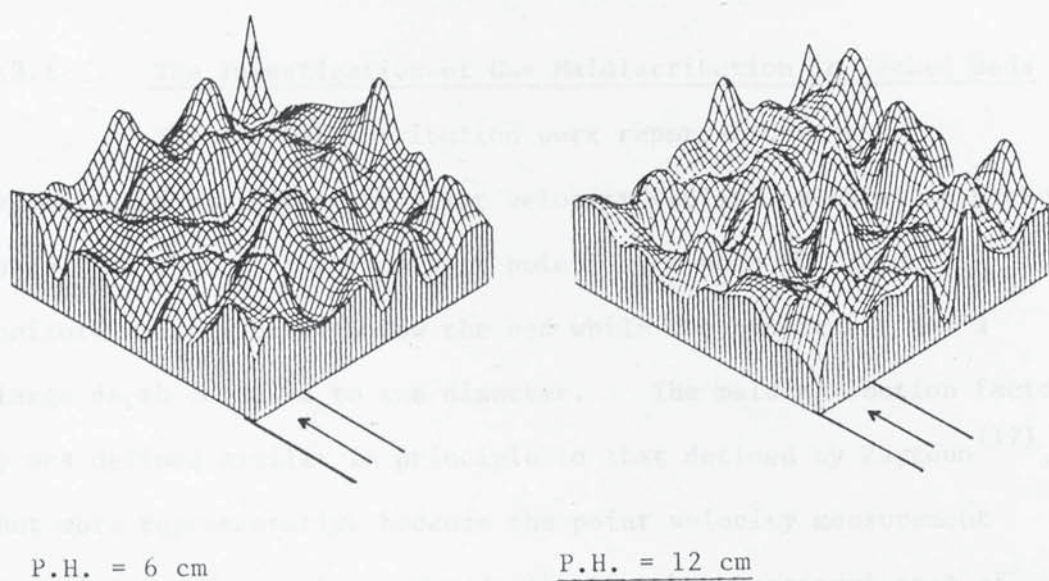


P.H. = 6 cm



P.H. = 12 cm

Fig. 11.9    Modification C Associated with  
Draw-off Tray



## SECTION XII

### DISCUSSION

It is known that the maldistribution of either gas or liquid may be expected to reduce the mass transfer performance of large diameter packed columns. All previous workers have studied the liquid maldistribution, and this work is believed to be the first extensive study of maldistributed gas flow in packed columns.

#### 12.1 The Investigation of Gas Maldistribution in Packed Beds

The gas maldistribution work reported here has established that point to point velocity variations exist in packed beds, and that is the important point, even when the feed was uniformly distributed below the bed while the bed itself has a large depth compared to the diameter. The maldistribution factor  $\phi$  was defined similar in principle to that defined by Zaytoun<sup>(17)</sup>, but more representative because the point velocity measurement technique used here has covered all the cross-sectional area of the packed bed, as shown in Section V. The most important feature of  $\phi$  is that it has a minimum value,  $\phi_{\min}$ , of about 20 for a deep bed when a uniformly distributed feed enters the bottom of the bed, and has much larger values for maldistributed beds.  $\phi_{\min}$  represents the best possible gas distribution. When this is achieved it is reasonable to expect that large diameter towers will work as well as small diameter towers (assuming of course that the liquid distribution also is uniform). The value of  $\phi$  for a packed bed is changed each time the bed is repacked, and this means that  $\phi$  is not reproducible. For this reason, each bed was repacked several times, and each time  $\phi$  was evaluated and then an average value,  $\bar{\phi}$ ,



was obtained which is reproducible and considered as the most adequate measure of gas maldistribution in packed beds.

## 12.2      The Method of Evaluating Quality of Gas Distributors and the Scale-up Rules

A method of measuring the quality of gas distributors has been established in Section VI based on the variation of  $\phi$  values with packed height, such that a good distributor is that one which produces gas maldistribution nearest to the minimum at the shortest packed height. It is practically impossible to apply this method to test the quality of a gas distributor in a large scale packed tower. Thus a laboratory size model of the full-scale column is needed to evaluate the gas distribution quality in such a large scale column. It is obvious that there should be rules to be followed in the construction of any model. Such rules of scaling up or scaling down a packed column were not available. A large part of the work was devoted to establish these rules which state that gas flow patterns are similar in two exactly geometrically similar packed beds of different sizes if the Reynolds number value is kept constant in both beds providing, and this is the important point, that the Mach number value is low enough to consider any change in it as ineffective. In this work, the maximum value of Mach number did not exceed 0.1. Therefore, these scale-up rules are applicable in systems whose Mach number,  $M$ , equals 0.1 or less. Vacuum crude columns work at high Mach number in the feed pipe, thus they require more investigations. But nevertheless it is expected that the proposed gas distributors will be better than those used up to now.

The basis of obtaining the scale-up rules is to get similar  $\phi$  values for geometrically similar beds of different size operated at the same Re value. Thus the validity of  $\phi$  as a reliable measure needs to be confirmed, and that confirmation will be discussed next.

### 12.3 The Statistical Study of Gas Maldistribution in Packed Beds

The multi-factor analysis of the variance was used to provide more insight on the meaning of  $\phi$  and to prove the validity of scale-up rules. The choice of this statistical method was not only because it studies the maldistribution of the gas in more detail by producing numerical values to estimate the maldistribution in the spacial co-ordinates x and y, but because it studies the effect of increasing the size of the packed bed on the distribution of the gas while keeping Reynolds number constant. This study has concluded that the scale-up rules are valid, and that there are two constituents of  $\phi$ . The first constituent, which is constant in all situations of packed height, Reynolds number, uniformity or nonuniformity of the feed, etc., is due to the point to point velocity variation. The second constituent is due to the non-uniformity of the gas entering the bottom of the bed, and is referred to as the regional maldistribution, which varies significantly with the packed depth and the quality of the feed distribution.

Another technique was used in this thesis to visualize the velocity distribution of a gas emerging from the top of packed beds by producing response surfaces, which provides more evidence on the similarity of flow patterns in geometrically and dynamically similar beds and on the existence of the two sources of gas maldistribution; the point to point velocity variation and regional velocity variations.



#### 12.4 Fluid Mechanics of Gas Maldistribution in Packed Beds

Having observed and evaluated the gas maldistribution above packed beds, the need to know what happens inside a packed bed with a maldistributed feed arises at this stage.

Commonly, the feed is introduced to the column either tangentially or radially. The case of a tangential feed inlet was used to study the mechanics of maldistributed gas flow. The choice of this case was because of the simplicity of performing the experimental investigation due to the symmetry of gas flow below the bed. Such a symmetry permits the measurements to take place along one diameter only. The gas velocity distribution and pressure distribution below and inside the bed were measured in Section VIII. As far as is known, these measurements of pressure distribution in the presence of maldistributed flow are the first ever to be presented. It was shown that the swirling air below the bed generates high pressure in the peripheral area whose width is more or less the same as the diameter of feed pipe, and a low pressure in the middle area. The air is driven by the pressure difference laterally from the high pressure area to the low pressure area and axially to atmosphere. If there was enough packed height, then the pressure will homogenize across the packed bed, and consequently the gas flow; if the packed height is short, then a quantity of gas is driven out of the bed by the high pressure more than the feed inlet supplies. Thus a downwards flow of air and a negative pressure was observed in the low pressure area. The case of the radial feed inlet needs more investigation to know the exact mechanism of gas maldistribution.

#### 12.5 The Maldistributed Gas Flow Theoretical Models

Three models have been suggested to describe the

maldistributed gas flow in packed beds with tangential feed inlets. In the first model, the three-dimension model, a mass balance was performed on an elemental volume  $dx dy dz$  in the bed in the cases of a completely laminar flow, a completely turbulent flow, and a laminar and turbulent flow. In each of these cases a differential equation was produced. The solution of these equations will depend on a mathematical model which translates the velocity below the bed into pressure within the bed, which is not available yet. This is an obvious area for future work. Two simple models are presented; the two area model and the cross-flow model. These two models can only be an approximation of the actual flow process, since almost certainly lateral flow takes place at different levels inside the packed bed up to the level where the pressure profile is flat, and the width of the high pressure area and low pressure area is not constant throughout the packed height. In these two models the column designs subject to gas maldistribution could be identified, that is, the height of a packed bed below which the flow is severely maldistributed due to the appearance of downwards gas flows in the low pressure area. These two models have produced a flow pattern function of maldistributed flow through the packing which has the property of an asymptotic approach to uniform distribution with increasing packed height and this is the common characteristic of the experimental observations, which show that the maldistribution factor  $\phi$  decreases as the packed height increases to approach the best possible gas distribution represented by  $\phi_{\min}$ .

## 12.6 Existing and New Designs of Gas Distributors

The method of evaluating the quality of gas distribution devices proposed in Section VI was used in Section X to test some models of large-scale gas distributors. The construction of these models was governed by the scale-up rules obtained in this work. For this purpose, a real life pump around section in a 7.3m diameter vacuum crude fractionation column described in Section II was used as an example to construct a laboratory size model, 1.22m in diameter i.e. a scale-down ratio of 1/6 is used. A widely used internal cylinder gas distributor is tested in this work, and it is shown in Section X, that this distributor does little to improve the gas distribution, and that most of the improvement is due to the particular design of liquid draw-off tray used below the packed bed. It should be noted that the improvement in gas distribution with this particular design is to a large extent fortuitous and a result of the use of blank areas near the wall through which no gas can flow.

The thesis concludes by designing three new gas distribution devices, which are modifications of the Internal Cylinder. The test performed on them in Section XI shows that the internal cylinder with internal cross baffle is the best low pressure gas distributor that would be suitable for large diameter packed beds.

It should be pointed out that the new distributors were designed with the objective of removing the high velocity swirl which the pressure distribution work had shown to be the cause of the maldistribution. Of great practical interest is that the simple geometries used are such that the pressure drop through these new gas distributors is expected to be exceedingly small.

Also, it is believed that there will be no serious practical difficulties in building and installing them on an industrial scale.

#### 12.7 Future Work

The work described above might be usefully followed by several subsequent investigations. The value of development of a three dimensional theory has been mentioned above, but even the simple theoretical models suggest the need for experiments to determine the effect of inlet pipe to tower diameter ratio, and to investigate the way the variation of  $\phi$  with packed height depends on packing size. Also it should be noted that more work is needed on radial inlets, both experimental and theoretical. Although the new distributors have shown to be effective there is much to be done on optimizing their dimensions.

## SECTION XIII

### CONCLUSIONS

The main conclusions from this experimental work can be summarised as follows:

(1) A method is proposed for evaluating the quality of gas distributors used in large diameter packed columns. A good distributor is that one which gives a good gas distribution in very short depths of a large diameter packed bed.

(2) A maldistribution factor,  $\phi$ , was defined, which is related to the standard deviation by the number of measurements made. It has a minimum value for well distributed gas flow and much higher values for maldistributed gas flow. The average value of maldistribution factor,  $\bar{\phi}$ , is reproducible.

(3) It has been found that for two geometrically and dynamically similar systems of different sizes,  $\bar{\phi}$  has the same value. The principle of dynamic similarity is satisfied when both systems are operated at the same Reynolds number. That is true in both uniformly and maldistributed gas flows.

(4) A laboratory study was carried out on a scaledown model, exactly geometrically similar to a vacuum crude fractionation tower 7.3m in diameter with a packed depth of less than 2m, but 1/6 the size. Four types of gas distributors have been tested. These are:- The previously known Internal Cylinder associated with tangential flow distributor, and three modifications:

- A) Internal Cylinder with Slots and Cross Baffles.
- B) Internal Cylinder with Guides in the Annulus.
- C) Internal Cylinder with Internal Cross Baffles.



The internal cylinder with internal cross baffles has been shown to be an excellent method of achieving a uniform gas distribution with a negligible pressure drop.

Also, it has been noticed that the particular design of draw-off tray used in the large scale tower helps in distributing the gas significantly.

5) The gas maldistribution in packed beds is associated with significant radial pressure gradients. These have been measured and shown to be associated with the velocity of the gas in the feed pipe.

6) Two simple theoretical models of gas maldistribution in packed beds have been proposed to describe the flow of gas through packed beds, and to support the use of geometrically and dynamically similar models to investigate gas distribution and distributors in full-scale plants. These two models identify the packed bed designs "at risk" of gas maldistribution.

7) The effect of the ratio of inlet pipe/tower diameter is expected to be a significant variable in determining gas distribution.

## APPENDIX A1

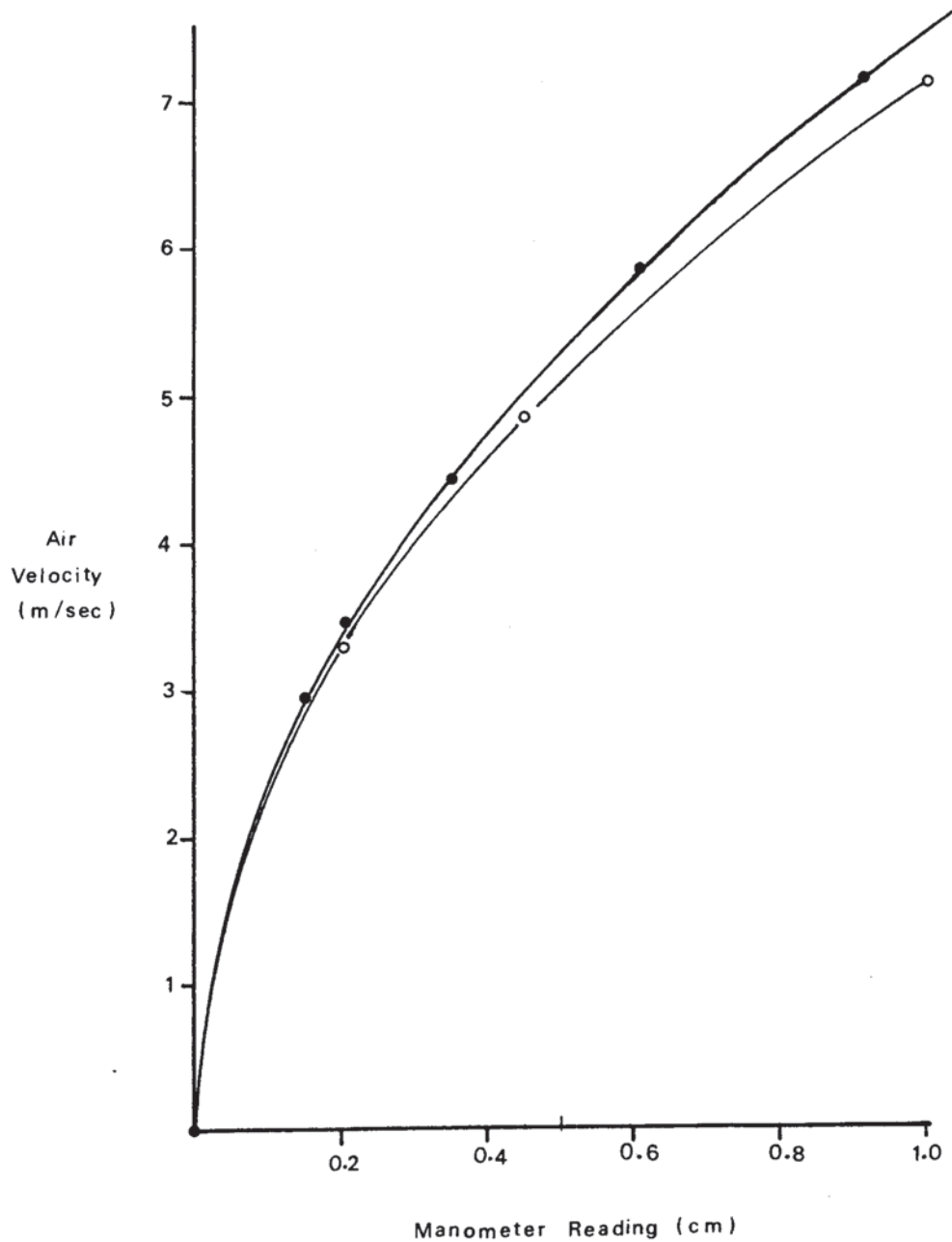
### Calibration Chart for the 152 mm Diameter Dall-Tube

Graph A1.1: Low air flow rates.

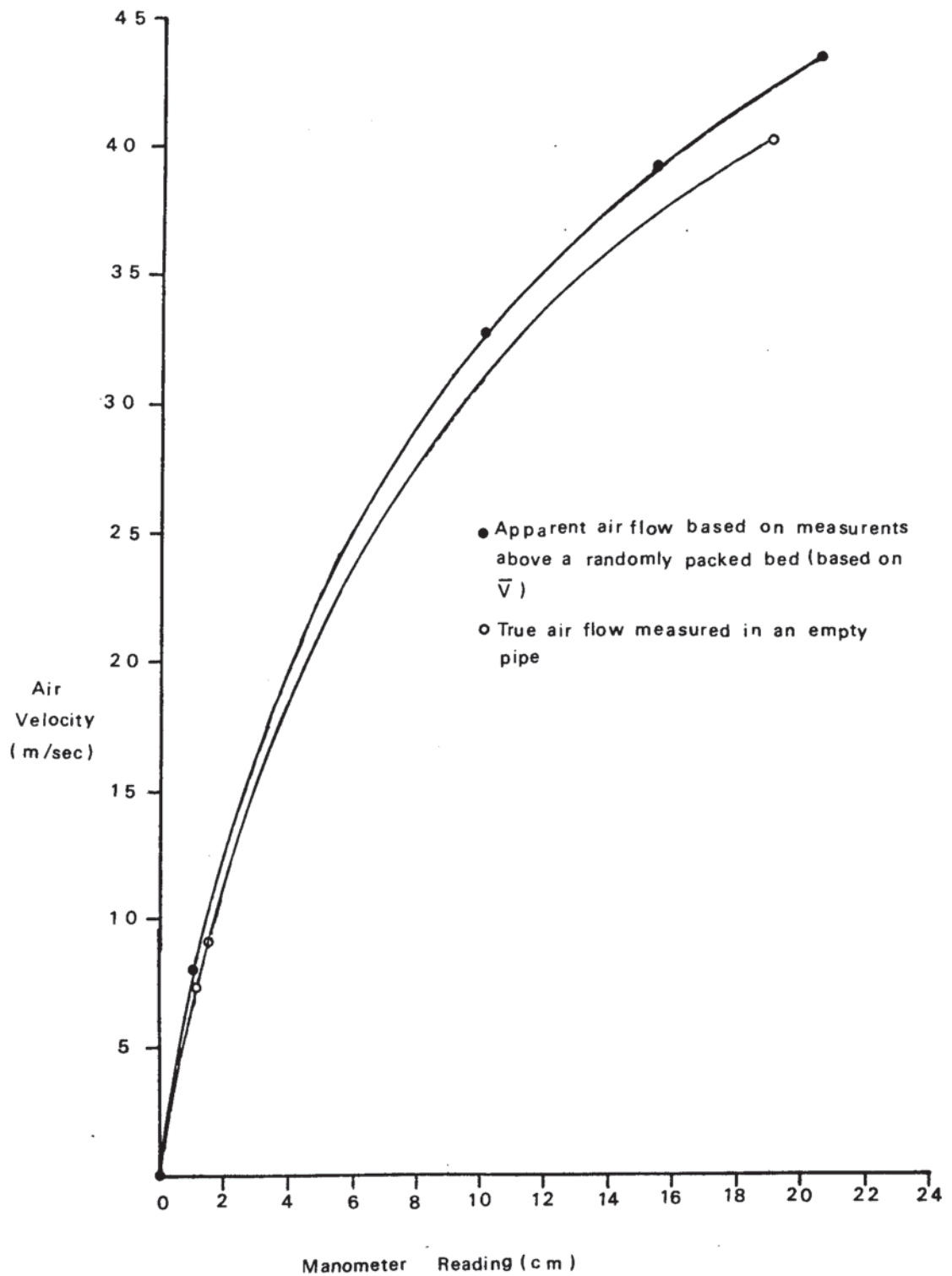
Graph A1.2: High air flow rates.

Graph A1.1 Calibration Chart for Low Air Flows

- Apparent air flow based on measurements above a randomly packed bed (based on  $\bar{V}$ )
- True air flow measured in an empty pipe



Graph A1.2 Calibration Chart for High Air Flows



APPENDIX A2

A COMPUTER PROGRAM TO CALCULATE  $\bar{V}$  and  $\phi$ , (F),  
FOR n POINT VELOCITIES

```
REAL V(n), VAV, SUM1, F
INTEGER k
READ*, (V(I), I = 1,n)
SUM1 = 0.0
DO 10 I = 1,k
SUM1 = SUM1 + V(I)
10 CONTINUE
VAV = SUM1/k
PRINT*, 'VAV =', VAV
F = 0.0
DO 20 I = 1,k
F = F + (1-(V(I)/VAV)**2
20 CONTINUE
PRINT*, 'F =', F
```

Evaluating  $\bar{V}$  (VAV) where  
$$\bar{V} = \frac{\sum V_i}{n}$$

Evaluating  $\phi$ , (F), where  
$$\phi = \sum \left(1 - \frac{V}{\bar{V}}\right)^2$$



## APPENDIX A3

### POINT VELOCITY MEASUREMENTS ABOVE GEOMETRICALLY AND DYNAMICALLY SIMILAR BEDS

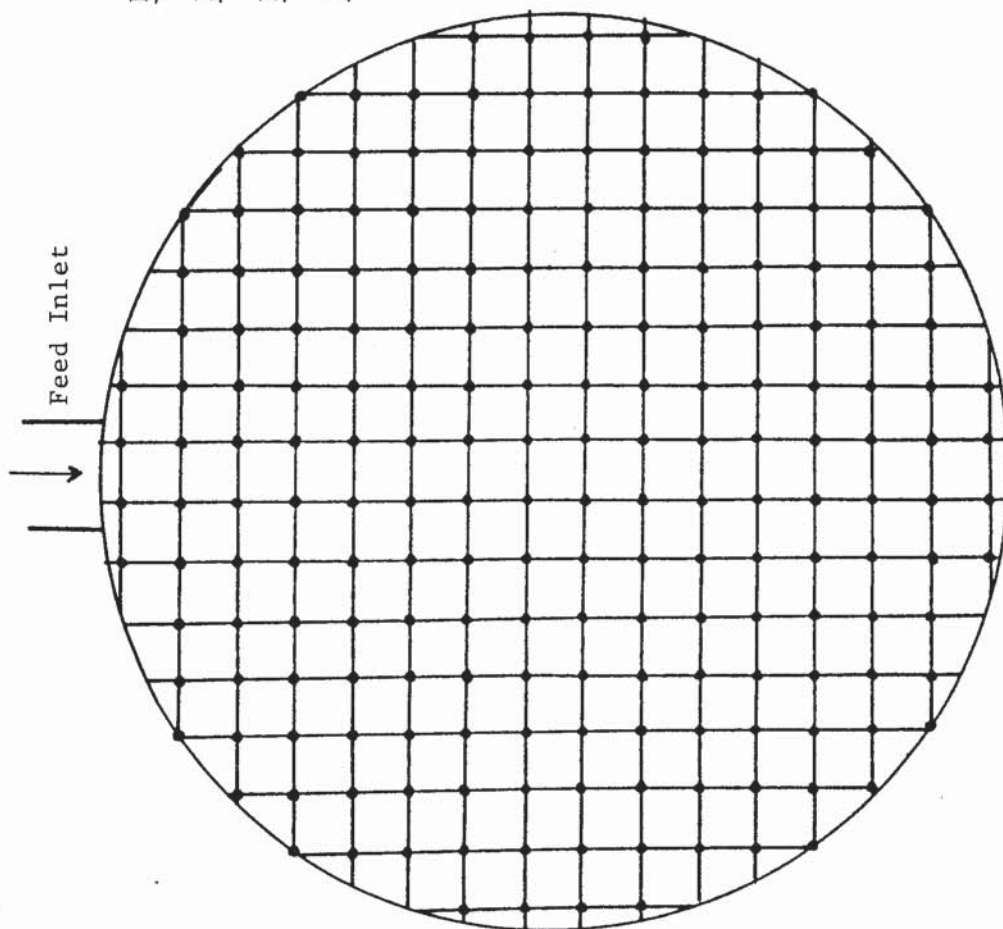
Tables A3.1 to A3.7:	D = 381 mm
	P.H. = several values
	Re = several values
	Maldistributed feed
Tables A3.8 to A3.13:	D = 610 mm
	P.H. = several values
	Re = several values
	Maldistributed feed
Tables A3.14 to A3.20:	D = 381 mm
	P.H. = several values
	Re = several values
	Ideally distributed feed
Tables A3.21 to A3.26:	D = 610 mm
	P.H. = several values
	Re = several values
	Ideally distributed feed

Key Figure A3.1

Positions of the 200 Point Velocity

Measurement in Tables (A3.1) to

(A3.26)





















[illegible]

Table A3.8 Point Velocity Measurements; Maldistributed, D = 610 mm, P.H. = 101.6 mm, P.S. = 25.4 mm

$$\bar{V} = 0.5 \text{ m/sec.}, \quad \text{Re} = 18000$$

$$\phi = 73.2$$

[illegible]

Table A3.9 Point Velocity Measurements; Maldistributed, D = 610 mm, P.H. = 101.6 mm, P.S. = 25.4 mm

$$\bar{V} = 0.60 \text{ m/sec.}, \quad \text{Re} = 25000$$

$$\phi = 81.65$$





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Table A3.11

Point Velocity Measurements ; Maldistributed, D = 610 mm, P.H. = 203 mm, P.S. = 25.4 mm

$$\bar{V} = 0.49 \text{ m/sec.}, \quad Re = 18000$$

$$\phi = 27.0$$







[illegible]

**Table A3.14** Point Velocity Measurements; Ideally distributed,  $D = 381$  mm, P.H. = 63.5 mm, P.S. = 16 mm

$$\bar{V} = 0.6 \text{ m/sec, } Re = 16150$$

$$\emptyset = 19.45$$







Table A3.17

$$\bar{V} = 0.6 \text{ m/sec, } Re = 16150$$
$$\phi = 21.84$$
[illegible]









$$\bar{v} = 0.466 \text{ m/sec}, \quad \text{Re} = 20000$$
$$\phi = 20.28$$
[illegible]

125	140	125	135	100	140	110	125	125	130	105	105	90
125	60	60	85	140	100	110	160	110	110	110	40	125
90	105	130	75	65	80	120	170	130	90	120	55	120
40	120	150	140	140	105	150	145	140	100	85	155	160
	95	160	110	135	90	120	130	70	35	120	110	200
	175	125	170	165	140	135	50	125	160	60	135	150
	145	180	120	120	145	140	165	160	210	95	140	120
		140	90	100	90	75	90	155	125	180	130	
			75	70	120	150	90	45	125	150		
						135	25	80	30			

Table A3.21 Point Velocity Measurements (ft/min); Ideally distributed, D = 610 mm, P.H. = 101.6 mm, P.S. = 25.4 mm

$$\bar{V} = 0.604 \text{ m/sec, } Re = 26000$$

$$\phi = 19.72$$







50	140	100	100	135	105	110	120	130	55	115	100	90	85	110	115	120	160	160	280	240	100	120	150
185	150	115	90	115	110	125	90	115	105	125	125	70	85	125	170	170	100	160	70	125	185	170	65
55	140	100	144	120	120	95	100	100	170	105	145	85	70	130	145	100	160	70	130	155	170	170	170
120	140	155	115	90	85	70	100	100	95	35	155	115	70	100	100	70	100	55	125	180	115	110	110
	90	110	95	55	170	125	80	95	95	100	120	125	160	180	125	160	180	125	125	180	115	110	110
	140	100	100	120	120	105	105	105	140	150	95	155	135	125	180	125	160	180	125	180	115	110	110
	115	140	110	110	135	80	110	85	115	115	145	70	75	140	115	115	70	75	140	115	110	110	110
		60	70	135	180	195	200	135	120	120	140	35	40	130									
			100	55	140	150	110	155	70	135	200	180											
						30	95	35	75														

Table A3.24: Point Velocity Measurements (ft/min); Ideally distributed, D = 610 mm, P.H. = 203 mm, P.S. = 25.4 mm  
 $\bar{V} = 0.6$  m/sec, Re = 26,000,  $\phi = 20.13$



[illegible]

Table A3.25. Point Velocity Measurements (ft/min); Ideally distributed, D = 610 mm, P.H. = 203 mm, P.S. = 25.4 mm  
 $\bar{V} = 0.66$  m/sec, Re = 29,000,  $\phi = 21.3$ .

## APPENDIX A4

### MULTI-FACTOR ANALYSIS OF THE VARIANCE

(Adopted from "Probability and Statistics for Engineers", by  
IRWIN MILLER and JOHN E. FREUND)

#### A4.1 Two-Factor Analysis of the Variance

If (a) point velocity measurements, "levels" are taken along x-axis, "factor X", and (b) point velocity measurements are taken along y-axis, "factor Y", then the xy plane consists of (ab) points, each point is specified by its xy-co-ordinates and has its specific gas velocity (point velocity).

The co-ordinates of these (ab) points will be as follows:

X: 1,1,.....,1,2,2,.....,2,3,3,.....3,.....a

Y: 1,2,.....b,1,2,.....b,1,2,.....b,.....b

And if several repackings, "replicates" were made, each consists of (ab) point velocity measurements, then it would be possible to analyse the data as a two-factor classification and test for significant differences among the (ab) point velocity means.

It is useful to make these remarks:

a = The number of point measurements, "levels", made along  
x-axis.

b = The number of point measurements, "levels", made along  
y-axis.

r = The number of bed repackings, "replicates".

ab = The number of point velocity measurements,  
"observations", made at each repacking.

rab = Total number of point velocity measurements,  
"observations" made at each experiment.

Note that the (ab) observations were chosen in such a way

that each level of x-axis is used once in conjunction with each level of y-axis.

In order to obtain an estimate of the experimental error in the experiment it is necessary to replicate, that is, to repeat the entire set of (ab) velocity measurements after repacking the bed, say, r times.

If  $(Y_{ijk})$  is the gas velocity in the (Kth) replicate, taken at (ith) level of x-axis and (jth) level of y-axis, the model assumed for the analysis of this kind of experiment is usually written as:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \rho_k + \epsilon_{ijk} \dots (A4.1)$$

For  $i = 1, 2, 3, \dots, a$ ,  $j = 1, 2, 3, \dots, b$ ,  $k = 1, 2, 3, \dots, r$

$\mu$  = The grand mean "average velocity of all repackings".

$\alpha_i$  = The effect of the (ith) point velocity along the x-axis on the distribution of the gas.

$\beta_j$  = The effect of the (jth) point velocity along the y-axis on the distribution of the gas

$(\alpha\beta)_{ij}$  = The interaction, or the joint effect, of (ith) level of x-axis and (jth) level of y-axis on the gas distribution.

$\rho_k$  = The effect of the (kth) replicate on the gas distribution

$\epsilon_{ijk}$  = The values of independent random variables having normal distribution with zero means and the common variance  $\sigma^2$ .

#### A4.1.1 Mathematical Formulation

The following symbols are defined as:

$T_{ij.}$  = Sum of observations in the (ij)th position, "cell".  
 $T_{i..}$  = Sum of observations for the (ith) level of factor X.  
 $T_{.j.}$  = Sum of observations for the (jth) level of factor Y.  
 $T_{...}$  = Sum of all (a.b.r) observations.  
 $\bar{Y}_{ij.}$  = Mean of observations in the (ij)th cell.  
 $\bar{Y}_{i..}$  = Mean of observations for the (ith) level of factor X.  
 $\bar{Y}_{.j.}$  = Mean of observations for the (jth) level of factor Y.  
 $\bar{Y}_{...}$  = Mean of all observations (a.b.r).

The (abr) observations are arranged in Table A4.1.

The hypotheses to be tested are as follows:

- (1) For a uniformly distributed gas emerging from the top of a packed bed:

$$H'_O : \alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha_a = 0 \quad \dots (A4.2a)$$

$$H''_O : \beta_1 = \beta_2 = \beta_3 = \dots = \beta_b = 0 \quad \dots (A4.2b)$$

$$H'''_O : (\alpha\beta)_{11} = (\alpha\beta)_{12} = \dots = (\alpha\beta)_{21} = (\alpha\beta)_{22} = \dots \\ = (\alpha\beta)_{ab} = 0 \quad \dots (A4.2c)$$

$$H''''_O : \rho_1 = \rho_2 = \rho_3 = \dots = \rho_r = 0 \quad \dots (A4.2d)$$

- (2) For non-uniformly distributed gas emerging from the top of a packed bed:

At least one of the  $\alpha_i$ 's or  $\beta_j$ 's or  $(\alpha\beta)_{ij}$ 's or  $\rho_k$ 's is not equal to zero.

Each of these tests will be based on a comparison of independent estimates of the variance,  $\sigma^2$ , provided by splitting of the total sum of squares of the experimental data into five components by means of the following identity:

x \ y	y					Total	Mean
	1	2	3	.....	b		
1	$Y_{111}$	$Y_{121}$	$Y_{131}$	.....	$Y_{1b1}$	$T_{1..}$	$\bar{Y}_{1..}$
	$Y_{112}$	$Y_{122}$	$Y_{132}$	.....	$Y_{1b2}$		
	$Y_{113}$	$Y_{123}$	$Y_{133}$	.....	$Y_{1b3}$		
	.	.	.		.		
	.	.	.		.		
2	$Y_{11r}$	$Y_{12r}$	$Y_{13r}$	.....	$Y_{1br}$	$T_{2..}$	$\bar{Y}_{2..}$
	$Y_{211}$	$Y_{221}$	$Y_{231}$	.....	$Y_{2b1}$		
	$Y_{212}$	$Y_{222}$	$Y_{232}$	.....	$Y_{2b2}$		
	$Y_{213}$	$Y_{223}$	$Y_{233}$	.....	$Y_{2b3}$		
	.	.	.		.		
.	$Y_{21r}$	$Y_{22r}$	$Y_{23r}$	.....	$Y_{2br}$	$T_{a..}$	$\bar{Y}_{a..}$
	.	.	.		.		
	.	.	.		.		
	.	.	.		.		
	.	.	.		.		
a	$Y_{a11}$	$Y_{a21}$	$Y_{a31}$	.....	$Y_{ab1}$	$T_{a..}$	$\bar{Y}_{a..}$
	$Y_{a12}$	$Y_{a22}$	$Y_{a32}$	.....	$Y_{ab2}$		
	.	.	.		.		
	.	.	.		.		
	.	.	.		.		
.	$Y_{alr}$	$Y_{a2r}$	$Y_{a3r}$	.....	$Y_{abr}$	$T_{a..}$	$\bar{Y}_{a..}$
	.	.	.		.		
	.	.	.		.		
	.	.	.		.		
	.	.	.		.		
Total	$T_{.1.}$	$T_{.2.}$	$T_{.3.}$	.....	$T_{.b.}$	$T_{...}$	
Mean	$\bar{Y}_{.1.}$	$\bar{Y}_{.2.}$	$\bar{Y}_{.3.}$	.....	$\bar{Y}_{.b.}$		$\bar{Y}_{...}$



$$\begin{aligned}
\sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^r (Y_{ijk} - \bar{Y}_{...})^2 &= rb \sum_{i=1}^a (\bar{Y}_{i..} - \bar{Y}_{...})^2 \\
&+ ra \sum_{j=1}^b (\bar{Y}_{.j.} - \bar{Y}_{...})^2 \\
&+ r \sum_{i=1}^a \sum_{j=1}^b (\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})^2 \\
&+ ab \sum_{k=1}^r (\bar{Y}_{..k} - \bar{Y}_{...})^2 \\
&+ \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^r (Y_{ijk} - \bar{Y}_{ij.} - \bar{Y}_{..k} + \bar{Y}_{...})^2
\end{aligned}
\tag{A4.3}$$

The left hand term is referred to as "Total Sum of Squares", (SST), and the right hand terms are referred to as

Factor X Sum of Squares, SSX.

Factor Y Sum of Squares, SSY.

Interaction Sum of Squares, SS(XY).

Replicate Sum of Squares, SSR.

Error Sum of Squares, SSE.

Then equation A4.3 becomes

$$SST = SSX + SSY + SS(XY) + SSR + SSE \tag{A4.4}$$

where:

SST has  $(abr-1)$  degrees of freedom.

SSX has  $(a-1)$  degrees of freedom.

SSY has  $(b-1)$  degrees of freedom.

SS(XY) has  $(a-1)(b-1)$  degrees of freedom.

SSR has  $(r-1)$  degrees of freedom.

SSE has  $(ab-1)(r-1)$  degrees of freedom.

Dividing each of the sum of squares on the right hand side of eqn. A4.4 by their corresponding number of degrees of freedom results in five independent estimates of "Mean Squares" defined as

$$S_1 = \frac{SSX}{(a-1)} \quad \dots\dots (A4.5a)$$

$$S_2 = \frac{SSY}{(b-1)} \quad \dots\dots (A4.5b)$$

$$S_3 = \frac{SS(XY)}{(a-1)(b-1)} \quad \dots\dots (A4.5c)$$

$$S_4 = \frac{SSR}{(r-1)} \quad \dots\dots (A4.5d)$$

$$S = \frac{SSE}{(ab-1)(r-1)} \quad \dots\dots (A4.5e)$$

Then five independent estimates of the variance,  $\sigma^2$ , can be obtained.

If the sums of squares are interpreted as "functions" of the independent random variable  $y_{111}, y_{112}, \dots, y_{abr}$  it can be verified that

$$E(S_1^2) = E \left[ \frac{SSX}{(a-1)} \right] = \sigma^2 + \frac{rb \sum_{i=1}^a \alpha_i^2}{(a-1)} \quad \dots\dots (A4.6a)$$

$$E(S_2^2) = E \left[ \frac{SSY}{(b-1)} \right] = \sigma^2 + \frac{ra \sum_{j=1}^b \beta_j^2}{b-1} \quad \dots\dots (A4.6b)$$

$$E(S_3^2) = E \left[ \frac{SS(XY)}{(a-1)(b-1)} \right] = \sigma^2 + \frac{\sum_{i=1}^a \sum_{j=1}^b (\alpha\beta)_{ij}^2}{(a-1)(b-1)} \quad \dots\dots (A4.6c)$$

$$E(S_4^2) = E \left[ \frac{SSR}{(r-1)} \right] = \sigma^2 + \frac{\sum_{k=1}^r \rho_k^2}{(r-1)} \quad \dots\dots (A4.6d)$$

$$E(S^2) = E \left[ \frac{SSE}{(ab-1)(r-1)} \right] = \sigma^2 \quad \dots\dots (A4.6e)$$

To test the hypothesis  $H'_0$ , that is the effects of factor X are all equal to zero, the following ratio is computed:

$$f_1 = \frac{S_1^2}{S^2} \quad \dots\dots (A4.7a)$$

which is the value of the random variable  $F_1$  having the F-distribution with  $(a-1)$  and  $(ab-1)(r-1)$  degrees of freedom when  $H'_0$  is true. The above hypothesis, "null hypothesis", is rejected at the level of confidence  $(\alpha)$  when;

$$f_1 > f_\alpha [(a-1), (ab-1)(r-1)]$$

To test the hypothesis  $H''_0$ , that the effects of factor Y are all equal to zero, the ratio  $f_2$  is computed as follows

$$f_2 = \frac{S_2^2}{S^2} \quad \dots\dots (A4.7b)$$

which is the value of the random variable  $F_2$  having the F-distribution with  $(b-1)$  and  $(ab-1)(r-1)$  degrees of freedom when  $H''_0$  is true. This hypothesis is rejected at the  $(\alpha)$  level of confidence when

$$f_2 > f_\alpha [(b-1), (ab-1)(r-1)]$$

Similarly, to test  $H'''_0$  and  $H''''_0$ .

The computations in an analysis of variance problem, for a two-factor experiment with (r) replications, "repackings", are usually summarized in Table A4.2.

The sums of squares are usually obtained by constructing a table of totals, as shown in Table A4.3, and using the following computational formulas:

$$SST = \sum_{i=1}^b \sum_{j=1}^a \sum_{k=1}^r Y_{ijk}^2 - \frac{T_{...}^2}{abr} \quad \dots\dots (A4.8a)$$

$$SSX = \frac{\sum_{i=1}^a T_{1..}^2}{br} - \frac{T_{...}^2}{abr} \quad \dots\dots (A4.8b)$$

$$SSY = \frac{\sum_{j=1}^b T_{.j.}^2}{ar} - \frac{T_{...}^2}{abr} \quad \dots\dots (A4.8c)$$

$$SS(XY) = \frac{\sum_{i=1}^a \sum_{j=1}^b T_{ij.}^2}{r} - \frac{\sum_{i=1}^a T_{i..}^2}{br} - \frac{\sum_{j=1}^b T_{.j.}^2}{ar} + \frac{T_{...}^2}{abr} \quad \dots\dots (A4.8d)$$

$$SSR = \frac{\sum_{k=1}^r T_{...k}^2}{ab} - \frac{T_{...}^2}{abr} \quad \dots\dots (A4.8e)$$

$$SSE = SST - SSX - SSY - SS(XY) - SSR \quad \dots\dots (A4.8f)$$

Table A4.2 Analysis of Variance for the Two-Factor Experiment with (r)

Replications				
Source of Variance	Sum of Squares	Degrees of Freedom	Mean Squares	Computed f
Replication	SSR	(r-1)	$S_4^2 = \frac{SSR}{r-1}$	$f_4 = \frac{S_4^2}{S^2}$
Main effects: x	SSx	(a-1)	$S_1^2 = \frac{SSx}{a-1}$	$f_1 = \frac{S_1^2}{S^2}$
y	SSy	(b-1)	$S_2^2 = \frac{SSy}{b-1}$	$f_2 = \frac{S_2^2}{S^2}$
Interaction xy	SS(xy)	(a-1)(b-1)	$S_3^2 = \frac{SS(xy)}{(a-1)(b-1)}$	$f_3 = \frac{S_3^2}{S^2}$
Error	SSE	(ab-1)(r-1)	$S^2 = \frac{SSE}{(ab-1)(r-1)}$	
	SST	abr-1		

Table A4.3 Sum of Squares for the Two-Factor Experiment with (r)

Replications						
<div><div>x</div><div>y</div></div>	1	2	3		6	
1	$T_{11\cdot}$	$T_{12\cdot}$	$T_{13\cdot}$	.....	$T_{1b\cdot}$	$T_{1\cdot\cdot}$
2	$T_{21\cdot}$	$T_{22\cdot}$	$T_{23\cdot}$	.....	$T_{2b\cdot}$	$T_{2\cdot\cdot}$
3	$T_{31\cdot}$	$T_{32\cdot}$	$T_{33\cdot}$	.....	$T_{3b\cdot}$	$T_{3\cdot\cdot}$
.	.	.	.			.
.	.	.	.			.
.	.	.	.			.
.	.	.	.			.
a	$T_{a1\cdot}$	$T_{a2\cdot}$	$T_{a3\cdot}$		$T_{ab\cdot}$	$T_{a\cdot\cdot}$
	$T_{\cdot 1\cdot}$	$T_{\cdot 2\cdot}$	$T_{\cdot 3\cdot}$		$T_{\cdot b\cdot}$	$T_{\cdot\cdot\cdot}$



#### A4.1.2    Application of the Two-factor Analysis of the Variance

Example (1): A 305 mm diameter bed packed with 25.4 mm pall rings to a height of 51 mm, with maldistributed feed flowing at a rate of  $0.1 \text{ m}^3/\text{s}$ . The bed was repacked three times and the following point velocities were recorded at positions  $x_i y_j$ , where  $i=1,2,\dots,12$  and  $j=1,2,\dots,12$

Note: The point velocities were recorded in (ft/min) for more convenience.

A computer statistical package, (UA10), is used to perform the solution.

Location		Replication Number			Total
x	y	1	2	3	
1	1	50	45	40	135
1	2	45	45	55	145
1	3	45	55	60	160
1	4	45	50	70	165
1	5	45	50	50	145
1	6	100	95	75	270
1	7	140	150	200	490
1	8	110	90	200	400
1	9	150	60	100	310
1	10	90	70	75	235
1	11	40	65	60	165
1	12	35	40	40	115
2	1	125	70	60	255
2	2	65	50	65	180
2	3	50	45	60	155
2	4	50	40	55	145
2	5	65	75	45	185
2	6	70	100	60	230
2	7	90	120	230	440
2	8	120	100	230	450
2	9	140	55	150	345
2	10	140	50	70	260
2	11	50	45	50	145
2	12	50	45	55	150
3	1	90	85	60	235
3	2	70	50	55	175
3	3	55	45	60	160
3	4	55	60	50	165
3	5	70	100	50	220
3	6	100	120	65	285
3	7	140	110	120	370
3	8	120	120	200	440
3	9	110	55	140	305

Location		Replication Number			Total
x	y	1	2	3	
3	10	80	35	75	190
3	11	60	40	70	170
3	12	45	50	70	165
4	1	120	125	120	365
4	2	75	55	75	205
4	3	50	50	100	200
4	4	50	70	75	195
4	5	70	135	85	290
4	6	100	110	90	300
4	7	150	100	125	375
4	8	120	100	140	360
4	9	90	75	190	355
4	10	75	75	140	290
4	11	65	60	100	225
4	12	85	70	140	295
5	1	140	220	170	530
5	2	120	75	120	315
5	3	65	70	130	265
5	4	40	75	115	230
5	5	70	175	160	405
5	6	85	160	160	405
5	7	130	90	160	380
5	8	160	75	220	455
5	9	110	140	200	450
5	10	55	120	160	335
5	11	65	100	160	325
5	12	150	85	160	395
6	1	125	270	160	555
6	2	130	230	150	510
6	3	75	100	75	250
6	4	50	50	200	300
6	5	60	140	240	440
6	6	105	190	220	515

Location		Replication Number			Total
x	y	1	2	3	
6	7	200	140	120	460
6	8	245	110	220	575
6	9	180	110	200	490
6	10	70	140	160	370
6	11	70	120	290	480
6	12	120	90	230	440
7	1	220	300	270	790
7	2	280	350	170	800
7	3	250	245	80	575
7	4	190	85	220	495
7	5	160	100	170	430
7	6	175	310	220	705
7	7	240	320	200	760
7	8	300	250	220	770
7	9	250	160	170	580
7	10	170	190	180	546
7	11	150	220	420	790
7	12	240	135	500	875
8	1	230	260	350	840
8	2	280	680	170	1130
8	3	450	480	50	980
8	4	500	300	140	940
8	5	350	270	200	820
8	6	280	350	150	780
8	7	420	400	150	970
8	8	450	240	165	855
8	9	420	200	350	970
8	10	400	250	320	970
8	11	320	420	430	1170
8	12	350	500	450	1300
9	1	60	345	480	885
9	2	110	480	80	670
9	3	420	250	80	750

Location		Replication Number			Total
9	4	500	200	130	830
9	5	520	210	350	1080
9	6	490	350	220	1060
9	7	710	180	230	1120
9	8	625	100	320	1045
9	9	420	240	340	1000
9	10	270	370	380	1020
9	11	450	450	300	1200
9	12	530	550	330	1410
10	1	130	245	500	875
10	2	210	500	220	930
10	3	630	200	400	1230
10	4	550	175	290	1015
10	5	400	200	335	935
10	6	580	250	250	1080
10	7	550	380	500	1430
10	8	800	180	620	1600
10	9	566	585	350	1495
10	10	370	450	450	1270
10	11	670	580	360	1610
10	12	850	760	330	1940
11	1	375	550	265	1190
11	2	200	320	260	780
11	3	450	150	570	1170
11	4	200	620	190	1010
11	5	295	400	130	825
11	6	800	500	720	2020
11	7	610	500	780	1890
11	8	165	700	700	1565
11	9	110	600	500	1210
11	10	130	400	600	1130
11	11	520	550	630	1700
11	12	630	850	870	2350

Location		Replication number			Total
x	y	1	2	3	
12	1	80	100	50	230
12	2	320	300	400	1020
12	3	560	400	650	1610
12	4	620	600	285	1505
12	5	760	270	310	1340
12	6	1510	1300	600	3410

Location		Replication number			Total
x	y	1	2	3	
12	7	1050	600	1050	2700
12	8	230	600	870	1700
12	9	90	550	260	900
12	10	170	400	260	830
12	11	600	450	230	1280
12	12	490	420	300	1210

$$\therefore \sum_{i=1}^{12} \sum_{j=1}^{12} \sum_{k=1}^3 Y_{ijk}^2 = 44489175 \text{ (sum of squares of all observations)}$$

$$\sum_{i=1}^{12} \sum_{j=1}^{12} T_{ij}^2 = 119290875 \text{ (sum of squares of the totals)}$$

$$T_{...}^2 = (103775)^2 = 1.076925 \times 10^{10} \text{ (square of the sum of the totals)}$$

$$\sum_{k=1}^r T_{..k}^2 = (36020)^2 + (33955)^2 + (33800)^2 \text{ (sum of the squares of sums of observations at each replication)}$$

The following table of totals is formed:



$\begin{matrix} y \\ x \end{matrix}$	1	2	3	4	5	6	7	8	9	10	11	12	TOTAL
1	135	145	160	165	145	270	490	400	310	235	165	115	2735
2	255	180	155	145	185	230	440	450	345	260	145	150	2940
3	235	175	160	165	220	285	370	440	305	190	170	165	2880
4	365	205	200	195	290	300	375	360	355	290	225	295	3455
5	530	315	265	230	405	405	380	455	450	335	325	395	4490
6	555	510	250	300	440	515	460	575	490	370	480	440	5385
7	790	800	575	495	430	705	760	770	580	540	790	875	8110
8	840	1130	980	940	820	780	970	855	970	970	1170	1300	11725
9	885	670	750	830	1080	1060	1120	1045	1000	1020	1200	1410	12070
10	875	930	1230	1015	935	1080	1430	1600	1495	1270	1610	1940	15410
11	1190	780	1170	1010	825	2020	1890	1565	1210	1130	1700	2350	16840
12	230	1020	1610	1505	1340	3410	2700	1700	900	830	1280	1210	17735
TOTAL	6885	6860	7505	6995	7115	11060	11385	10215	8410	7440	9260	10645	103775

$$\therefore \sum_{i=1}^{12} T_{i..}^2 = 1270030125 \text{ sum of squares of all observations in}$$

(ith) level of factor X

$$\sum_{j=1}^{12} T_{.j.}^2 = 931774475 \text{ sum of squares of all observations in the}$$

(jth) level of factor Y

Applying equations (8) to (13) where  $a = 12$ ,  $b = 12$ ,  $r = 3$ .

SST = 19560354	"Total sum of squares".
SSX = 10349794	"factor-X sum of squares".
SSY = 953803	"factor-Y sum of squares".
SS(XY) = 3531207	"Interaction sum of squares".
SSR = 21335	"Relication sum of squares"
SSE = 4704215	"Error sum of squares"

The computed f-values are as follows:

Source of Variation	Degrees of freedom	sum of squares	Mean Squares	Computed f
Replication	$3 - 1 = 2$	21335	10667.5	0.6485
X	$12 - 1 = 11$	10349754	940890	57.203
Y	$12 - 1 = 11$	953803	86709	5.272
Interaction	$(12-1)(12-1)=121$	3531207	29183.5	1.7743
Error	286	4704215	16448.3	
Total	431			

Comparing the computed  $f$  values with the tabulated values of  $(F)$  at significance level of  $\alpha = 0.05$  found in Table A4.4 under the appropriate degrees of freedom. The following results have been achieved

(1) For replication:  $F_{0.05} (2, 286) = 3$  (from Table 3A1 at  $\nu_1 = 2$  degrees of freedom and  $\nu_2 = 286$  degrees of freedom).

Compare the computed value of  $f_1 = 0.6485$

$$\therefore F_{0.05} (2, 286) = 3 > 0.6485$$

$\therefore$  Accept the hypothesis that replication or repacking has no significant effect on the whole picture of the gas distribution.

(2) For the effect of factor-X:  $F_{0.05} (11, 286) = 1.79$  (from Table 3A1)  $1.79 < 57.203$

$\therefore$  reject the hypothesis that factor-X has no effect.

i.e. the gas is severely maldistributed along x-direction, which is the direction of the feed entering the column below the bed.

(3) Factor-Y effect:  $F_{0.05} (11, 286) = 1.79 < 5.272$

$\therefore$  Gas distribution is distributed for less extent along y-direction.

(4) Interaction effect:  $F_{0.05} (121, 286) = 1.22 < 1.7743$

$\therefore$  The combined effect of factor X and factor Y on the distribution is effective.

This means that the gas is severely maldistributed in the x-direction i.e. in the direction of feed below the packed bed. Also, it is maldistributed in the y-direction but to a smaller level. When the bed is repacked, the distribution as a whole remains unaffected despite some insignificant changes in the details of the distribution.



Table A4.4

VALUES OF  $F_{.05}$ 

$n$ = Degrees of freedom for denominator	$n_1$ = Degrees of freedom for numerator																			
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$	
1	161	200	210	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254	
2	18.50	19.00	19.20	19.20	19.30	19.30	19.40	19.40	19.40	19.40	19.40	19.40	19.40	19.50	19.50	19.50	19.50	19.50	19.50	
3	10.10	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.91	1.89	1.84	1.79	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00	

\* This table is reproduced from M. Merrington and C. M. Thompson, "Tables of percentage points of the inverted beta ( $F$ ) distribution," *Biometrika*, Vol. 33 (1943)

### Example 2

The same bed described in Example 1, but with uniformly distributed feed below the bed. The bed was repacked three times and the following point velocities were recorded.



Location		Replication Number			Total
x	y	1	2	3	
1	1	355	85	55	495
1	2	205	210	165	580
1	3	260	445	110	815
1	4	155	440	225	820
1	5	140	200	200	540
1	6	295	150	390	835
1	7	110	310	315	735
1	8	120	235	415	770
1	9	310	215	330	855
1	10	340	65	240	645
1	11	420	175	200	795
1	12	420	215	280	915
2	1	275	300	180	755
2	2	220	185	275	680
2	3	95	260	260	615
2	4	105	280	190	575
2	5	185	195	285	665
2	6	580	550	550	1680
2	7	450	500	425	1375
2	8	455	175	420	1050
2	9	375	225	265	865
2	10	330	410	175	915
2	11	320	300	130	750
2	12	250	340	390	980
3	1	225	180	245	650

Location		Replication Number			Total
x	y	1	2	3	
3	2	135	195	350	680
3	3	130	145	225	500
3	4	50	170	255	475
3	5	20	440	325	785
3	6	310	550	420	1280
3	7	455	360	510	1325
3	8	360	170	305	835
3	9	210	620	245	1075
3	10	110	400	240	750
3	11	115	370	290	775
3	12	170	115	390	675
4	1	410	190	285	885
4	2	320	190	230	740
4	3	325	200	280	805
4	4	195	160	335	690
4	5	190	350	290	830
4	6	500	430	235	1165
4	7	260	435	325	1020
4	8	350	375	170	895
4	9	340	470	210	1020
4	10	60	550	410	1020
4	11	445	480	415	1340
4	12	370	360	380	1110
5	1	345	295	230	870
5	2	380	350	100	830

Location		Replication Number			Total
x	y	1	2	3	
5	3	545	255	95	895
5	4	440	225	25	690
5	5	260	310	190	760
5	6	480	280	270	1030
5	7	375	480	395	1250
5	8	365	440	390	1195
5	9	340	395	480	1215
5	10	130	145	240	515
5	11	500	110	365	975
5	12	480	295	410	1185
6	1	390	155	70	615
6	2	225	230	270	725
6	3	405	165	160	730
6	4	420	175	260	855
6	5	220	400	190	810
6	6	95	265	85	445
6	7	260	350	160	770
6	8	460	440	320	1220
6	9	360	500	285	1145
6	10	450	385	330	1165
6	11	360	300	330	990
6	12	365	240	160	765
7	1	125	125	180	430
7	2	105	260	215	580
7	3	85	390	215	690

Location		Replication Number			Total
x	y	1	2	3	
7	4	240	400	335	975
7	5	235	410	290	935
7	6	175	220	160	555
7	7	240	205	220	665
7	8	210	300	320	830
7	9	265	130	335	730
7	10	385	360	325	1070
7	11	320	380	270	970
7	12	215	315	270	800
8	1	155	280	300	735
8	2	105	310	265	680
8	3	120	235	360	715
8	4	155	110	275	540
8	5	265	140	250	655
8	6	170	135	80	385
8	7	180	260	170	610
8	8	115	360	315	790
8	9	110	145	300	645
8	10	560	415	285	1260
8	11	470	570	195	1235
8	12	500	270	395	1165
9	1	430	350	150	930
9	2	325	200	180	705
9	3	240	120	235	595
9	4	200	210	420	830

Location		Replication Number			Total	Location		Replication Number			Total
x	y	1	2	3		x	y	1	2	3	
9	5	440	220	265	925	11	6	390	315	510	1215
9	6	270	150	100	520	11	7	150	480	440	1070
9	7	155	320	215	690	11	8	280	370	440	1090
9	8	115	370	305	790	11	9	395	200	475	1070
9	9	190	285	500	975	11	10	295	250	495	1040
9	10	165	115	440	720	11	11	80	260	420	760
9	11	435	250	190	875	11	12	70	160	175	405
9	12	330	400	350	1080	12	1	85	100	325	510
10	1	295	365	155	815	12	2	105	110	315	530
10	2	235	290	215	740	12	3	470	320	295	1085
10	3	280	200	325	805	12	4	215	370	235	820
10	4	185	200	380	765	12	5	250	305	290	845
10	5	440	275	365	1080	12	6	295	265	400	960
10	6	580	280	360	1220	12	7	375	195	270	840
10	7	275	475	475	1225	12	8	360	340	115	815
10	8	235	400	445	1080	12	9	325	335	105	765
10	9	315	70	460	845	12	10	395	330	115	840
10	10	125	120	440	685	12	11	115	220	310	645
10	11	340	295	275	910	12	12	190	230	350	770
10	12	140	180	130	450						
11	1	355	260	120	735						
11	2	495	80	145	720						
11	3	375	260	100	735						
11	4	190	390	320	900						
11	5	600	380	290	1270						

$$\therefore \sum_{i=1}^{12} \sum_{j=1}^{12} \sum_{k=1}^3 Y_{ijk}^2 = 40831200$$

$$\sum_{i=1}^{12} \sum_{j=1}^{12} T_{ij}^2 = 111134950$$

$$T_{...}^2 = (122010)^2$$

$$\sum_{k=1}^3 T_{..k}^2 = (40575)^2 + (40945)^2 + (40490)^2$$

The table of totals is formed as follows:



$\begin{matrix} y \\ x \end{matrix}$	1	2	3	4	5	6	7	8	9	10	11	12	TOTAL
1	495	580	815	820	540	835	735	770	855	645	795	915	8800
2	755	680	615	575	665	1620	1375	1050	865	915	750	980	10905
3	650	680	500	475	785	1280	1325	835	1075	750	775	675	9805
4	885	740	805	690	830	1165	1020	895	1020	1020	1340	1110	11520
5	870	830	895	690	760	1030	1250	1195	1215	515	975	1185	11410
6	615	725	730	855	810	445	770	1220	1145	1165	990	765	10235
7	430	580	690	975	935	555	665	830	730	1070	570	800	9230
8	735	680	715	540	655	385	610	750	645	1260	1235	1165	9415
9	930	705	595	830	925	520	690	790	975	720	875	1080	9635
10	815	740	805	765	1080	1220	1225	1080	845	685	910	450	10620
11	735	720	735	900	1270	1215	1070	1090	1070	1040	760	405	11010
12	510	530	1085	820	845	960	840	815	765	840	645	770	9425
TOTAL	8425	8190	8985	8935	10100	11290	11575	11360	11205	10625	11020	10300	122010



$$\therefore \sum_{i=1}^{12} T_i^2 \dots = 1249654250$$

$$\sum_{j=1}^{12} T_{.j}^2 \dots = 1257098550$$

Applying Eqs. A4.8a - A4.8f, where a = 12, b = 12, r = 3

$$SST = 6371848$$

$$SSx = 253266$$

$$SSy = 260052$$

$$SS(xy) = 1872313$$

$$SSR = 813$$

$$SSE = 3985404$$

The computed f values are as follows:

Source of Variation	Degree of Freedom	Sum of Squares	Main Squares	Computed f
Replication	2	813	406.5	0.03
Factor -x	11	25366	23024	1.65
Factor -y	11	260052	23641	1.7
Interaction	121	1872313	15474	1.11
Error	286	3985405	13935	
Total	431	6371848		

Comparing computed f-values with tabulated F-values at significance level  $\alpha = 0.05$  at appropriate degrees of freedom found in Table A4.4:

- 1) For replication:  $F_{0.05} (2, 286) = 3 > 0.03$   
 $\therefore$  Replication has no significant effect.
- 2) Factor X:  $F_{0.05} (11, 286) = 1.79 > 1.65$   
 $\therefore$  Gas distribution is not significantly effected along the x-direction.
- 3) Factor Y:  $F_{0.05} (11, 286) = 1.79 > 1.7$   
 $\therefore$  Gas distribution is not significantly changed along the y-direction.
- 4) Interaction:  $F_{0.05} (121, 286) = 1.22 > 1.11$   
 $\therefore$  The combined effect of x and y has no significant effect on gas distribution.

This means that there is no significant maldistribution in the gas emerging from the top of a packed bed whose feed is ideally distributed. Each time the bed is repacked, the distribution of gas emerging from the top of that bed is changed, but that change has no significant effect on the distribution as a whole.

#### A4.2 Three-Factor Analysis of the Variance

In this section, an experiment with three factors is considered. These three factors are the previously defined X and Y co-ordinates in the xy plane above the packed bed, and the diameter, D, of the packed bed. That is, the effects of x-axis, y-axis and the diameter scale on the distribution of the gas above geometrically similar beds of different size operated at similar Reynolds number, are tested.

These remarks should be made:

- a = The number of point velocity measurements,  
"levels" taken along x-axis
- b = The number of levels taken along y-axis
- c = The number of beds, (of different diameters), the  
point velocities are measured.
- r = The number of repackings made for each bed.

Assume that (r) observations are made for each of the (abc) treatments. That is, a similar number of repackings should be made for each bed.

The model for the three-factor experiment is given by:

$$Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{ijkl} \quad \dots (A4.9)$$

$i = 1, 2, \dots, a; j = 1, 2, \dots, b; k = 1, 2, \dots, c; l = 1, 2, \dots, r.$

$\gamma_i, \beta_j, \gamma_k$  = The effects of (ith) level, (jth) level and (kth) level of X, Y, and D respectively on the distribution of the gas above the geometrically similar beds of different size.

The general idea concerning this analysis is the same as that discussed for the two-factor analysis of the variance. The sum of squares is partitioned into eight terms, each representing a source of variation of the gas distribution above geometrically similar beds.

Proceeding directly to the computational procedure for obtaining the sums of squares in the three-factor analysis of the variance. The following notations should be made:

- $T_{....}$  = Sum of all (abcr) observations.  
 $T_{i...}$  = Sum of the observations for the (ith) level  
of factor X.  
 $T_{.j..}$  = Sum of the observations for the (jth) level  
of factor Y.  
 $T_{..k.}$  = Sum of the observations for the (kth) level  
of factor D.  
 $T_{ij...}$  = Sum of the observations for the (ith) level of  
factor X and the (jth) level of factor Y.  
 $T_{i.k.}$  = Sum of the observations for the (ith) level  
of factor X and the (kth) level of factor D  
 $T_{.jk.}$  = Sum of the observation for the (jth) level  
of factor Y and the (kth) level of factor D  
 $T_{ijk.}$  = Sum of the observations for the (ijk) th  
treatment combination

The sums of squares are computed as follows:

$$SST = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^c \sum_{l=1}^r Y_{ijkl}^2 - \frac{T_{....}^2}{abcr} \quad \dots\dots (A4.10a)$$

$$SSX = \frac{\sum_{i=1}^a T_{i...}^2}{bcr} - \frac{T_{....}^2}{abcr} \quad \dots\dots (A4.10b)$$

$$SSY = \frac{\sum_{j=1}^b T_{.j..}^2}{acr} - \frac{T_{....}^2}{abcr} \quad \dots\dots (A4.10c)$$

$$SSD = \frac{\sum_{k=1}^c T_{..k.}^2}{abr} - \frac{T_{....}^2}{abcr} \quad \dots\dots (A4.10d)$$

$$SS(XY) = \frac{\sum_{i=1}^a \sum_{j=1}^b T_{ij..}^2}{cr} - \frac{\sum_{i=1}^a T_{i...}^2}{bcr} - \frac{\sum_{j=1}^b T_{.j..}^2}{acr} + \frac{T_{....}^2}{abcr} \dots\dots (A4.10e)$$

$$SS(XD) = \frac{\sum_{i=1}^a \sum_{k=1}^c T_{i.k.}^2}{br} - \frac{\sum_{i=1}^a T_{i...}^2}{bcr} - \frac{\sum_{k=1}^c T_{..k.}^2}{abr} + \frac{T_{....}^2}{abcr} \dots\dots (A4.10f)$$

$$SS(YD) = \frac{\sum_{j=1}^b \sum_{k=1}^c T_{.jk.}^2}{ar} - \frac{\sum_{j=1}^b T_{.j..}^2}{acr} - \frac{\sum_{k=1}^c T_{..k.}^2}{abr} + \frac{T_{....}^2}{abcr} \dots\dots (A4.10g)$$

$$SS(XYD) = \frac{\sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^c T_{ijk.}^2}{r} - \frac{\sum_{i=1}^a \sum_{j=1}^b T_{ij..}^2}{cr} - \frac{\sum_{i=1}^a \sum_{k=1}^c T_{i.k.}^2}{br} - \frac{\sum_{j=1}^b \sum_{k=1}^c T_{.jk.}^2}{ar} + \frac{\sum_{i=1}^a T_{i...}^2}{bcr} + \frac{\sum_{j=1}^b T_{.j..}^2}{acr} + \frac{\sum_{k=1}^c T_{..k.}^2}{abr} - \frac{T_{....}^2}{abcr} \dots\dots (A4.10h)$$

$$SSE = SST - SSX - SSY - SSD - SS(XY) - SS(XD) - SS(YD) - SS(XYD) \dots\dots (A4.10i)$$



Table A4.5 Analysis of the Variance for the Three-factor Experiment  
with (r) replicates

Source of Variance	Sum of squares	Degree of Freedom	Mean Squares	Computed-f
Main Effects:				
X	SSX	(a-1)	$S_1^2$	$f_1 = S_1^2 / S^2$
Y	SSY	(b-1)	$S_2^2$	$f_2 = S_2^2 / S^2$
D	SSD	(C-1)	$S_3^2$	$f_3 = S_3^2 / S^2$
Two-factor Interaction				
XY	SS(XY)	(a-1)(b-1)	$S_4^2$	$f_4 = S_4^2 / S^2$
XD	SS(XD)	(a-1)(c-1)	$S_5^2$	$f_5 = S_5^2 / S^2$
YD	SS(YD)	(b-1)(c-1)	$S_6^2$	$f_6 = S_6^2 / S^2$
Three-factor Interaction				
XYD	SS(XYD)	(a-1)(b-1)(c-1)	$S_7^2$	$f_7 = S_7^2 / S^2$
Error	SSE	abc(r-1)	$S^2$	
Total	SST	abcr-1		

The computations in an analysis of the variance problem, for a three-factor experiment with (r) replications, are summarized in Table A4.5.

Following the same method of the two-factor analysis of the variance, the significance of the effect of each term listed in Table A4.5 on the distribution of the gas is estimated. The solution was performed using a computer statistical package, (UA 27).

#### A4.2.1 Application of the Three-factor Analysis of the Variance

In this work the distribution of the air emerging from the top of two geometrically and dynamically similar packed beds of different size is analysed when the two beds were fed by

1 - Ideally distributed air.

2 - Maldistributed air.

In this analysis point Reynolds numbers were used instead of the point velocities, and

a = 12 (point measurements along x-axis).

b = 12 (point measurements along y-axis).

c = 2 ( $D_1 = 381$  mm,  $D_2 = 610$  mm).

r = 3 (each bed is repacked three times).

∴ ab = 144 (point measurements made above each bed at each repacking).

abr = 432 (point measurements made above each bed).

abcr = 864 (the total number of point measurements).

Re = 30,000 (for the two beds).

#### Example 1: Ideally distributed feed.

The specification of the two geometrically similar beds, (bed number 1 and bed number 2), are as follows:

	<u>Bed number 1</u>	<u>Bed number 2</u>
Diameter, D (mm)	381	610
Packed Height, P.H. (mm)	127	203
Pack Size, P.S. (mm)	16	25.4
D/P.H.	3	3
D/P.S.	24	24
Re	30,000	30,000

The test is performed by using a computer statistical package ( UA 27 ) which computes f values. The results are listed in Table A4.6.

Example 2: Maldistributed feed.

The test was performed by the use of the statistical package ( UA 27 ) on the same beds above but with maldistributed feed. The results are listed in Table A4.6.

Table A4.6. Computed f-values for two geometrically similar beds operated under Re = 30,000

Source of Variance	Computed f-values		Tabulated f-values
	Ideally distributed	Maldistributed	
X	1.613	11.753	1.71
Y	1.315	2.16	1.71
D	0.543	0.568	3.84
XY	1.416	1.325	1.22
XD	1.314	1.617	1.71
YD	1.482	1.47	1.71
XYD	0.74	0.953	1.22

The most important factors which effect the distribution of the gas are: Factor X, Factor Y and Factor D.

The comparison of the computed-f values of these factors with their corresponding tabulated f-values at  $\alpha = 0.05$ , (obtained from Table A4.4 at appropriate degrees of freedom), shows:

- 1) In the case of "Ideally" distributed feed the gas emerging from the top of the two beds is well distributed since the effects of X and Y factors are not significant. The diameter of the bed has no significant effect on the distribution of the gas emerging from the top of the two beds since the computed f-value of the factor D, (0.543) is smaller than the tabulated value, (3.84).
- 2) In the maldistributed feed case, the results show a severe maldistribution in the gas above the two beds specially along the x-axis.

Again the size of the bed, (Diameter), has no significant effect on the distribution of the gas above the two beds.

∴ For two geometrically similar beds of different size, the flow pattern of the gas is similar provided that they are operated at the same Re value.

## APPENDIX A5

### POINT VELOCITY MEASUREMENTS ABOVE PACKED BEDS SUPPLIED BY EXISTING DESIGNS OF GAS DISTRIBUTION DEVICES

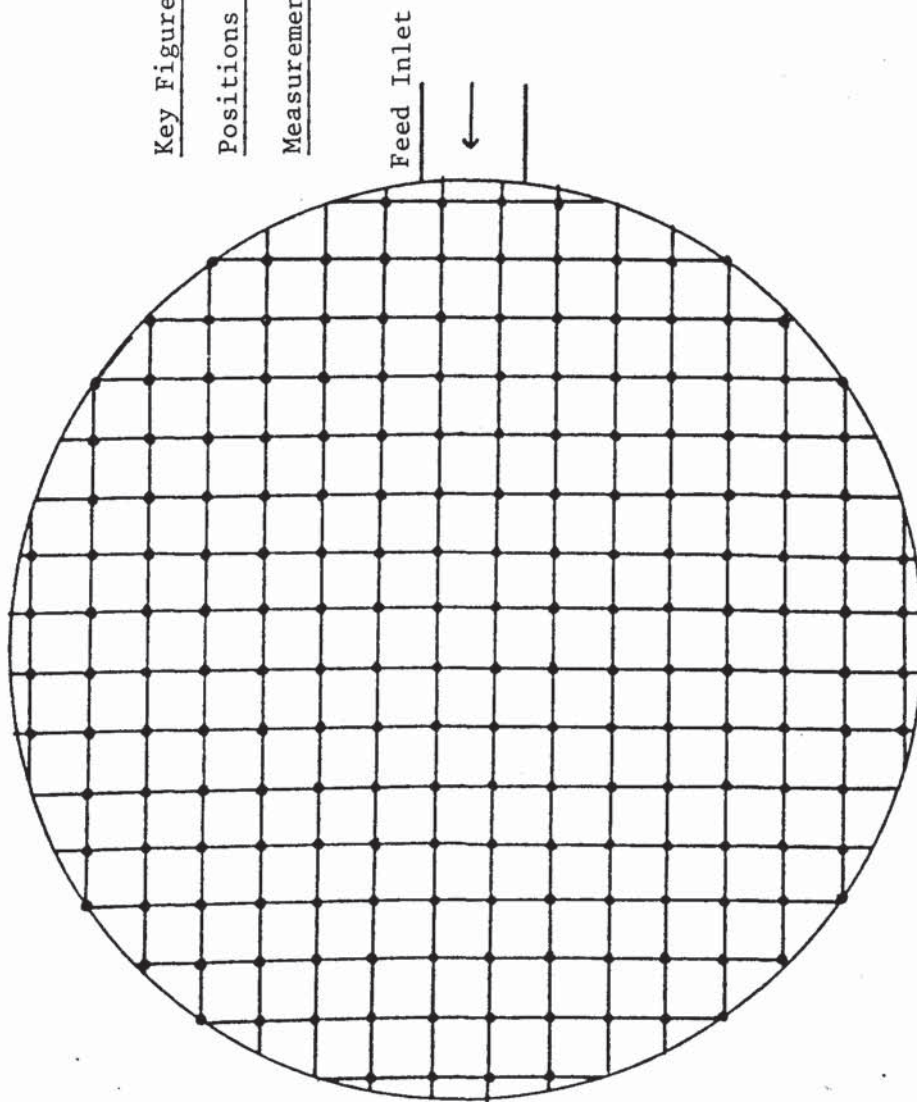
Tables A5.1 to A5.4:	Radial feed inlet, no distributor, several P.Hs.
Tables A5.5 to A5.8:	Radial feed inlet, draw-off tray, several P.Hs.
Tables A5.9 to A5.10:	Tangential feed inlet, no distributor, P.H. = 12.30 cm
Tables A5.11 to A5.12:	Tangential feed inlet, draw-off tray, P.H. = 6,12 cm.
Table A5.13:	Tangential feed inlet, Internal cylinder P.H. = 6 cm.
Computer program to draw the response surfaces.	



Key Figure A5.1

Positions of the 200 Point Velocity

Measurements in Tables A5.1 5o A5.8











1

Table A5.4 Point Velocity Measurements (ft/min); No Distributor, P.H. = 30 cm, Radial Inlet, P.S. = 16 mm

$$Re = 46000$$
$$\phi. = 21.4$$







[illegible]

Table A5.7 Point Velocity Measurements (ft/min); Draw-off Tray, P.H. = 12 cm, Radial Inlet P.S. = 16 mm

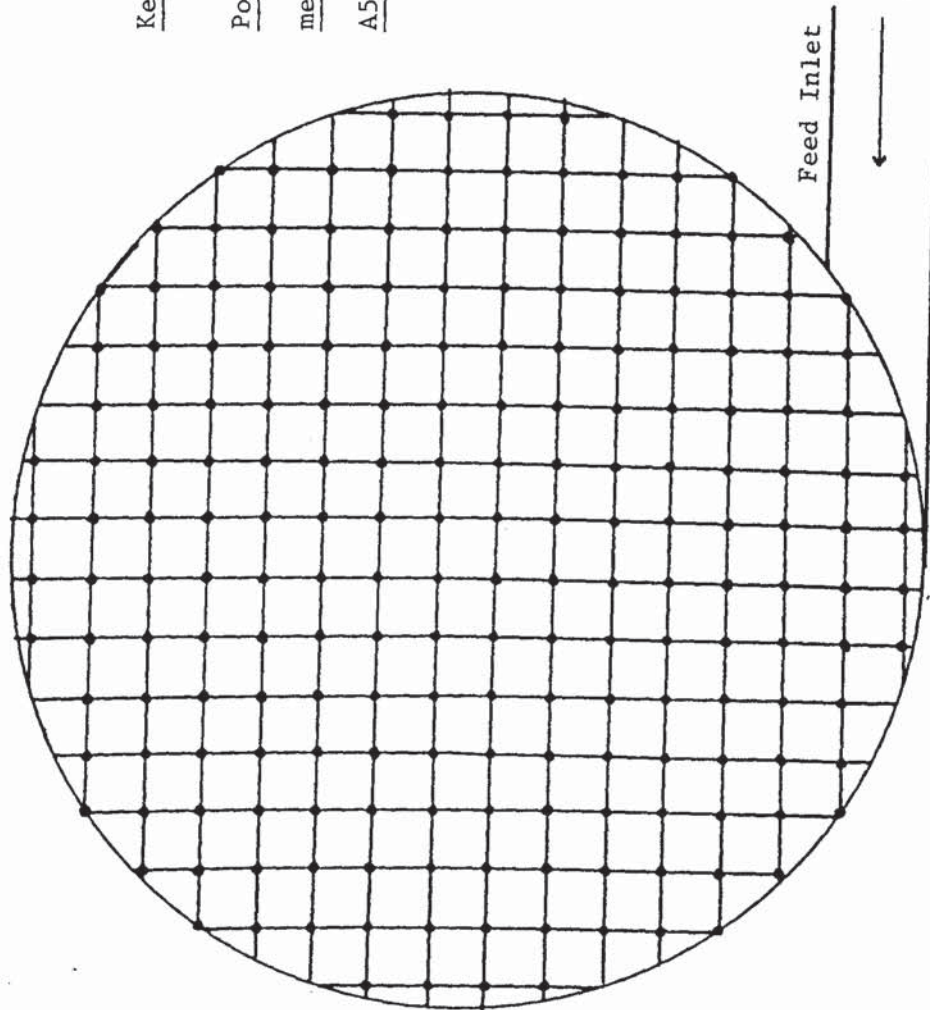
$$Re = 46000$$
$$\phi = 22.6$$





Key Figure A5.2

Positions of the 200 point velocity  
measurements in tables A5.9 to  
A5.13





$$\phi = 148.5$$



$$\text{Re} = 4600$$

$\phi = 60$

[illegible]



$$Re = 46000$$

$$= 38$$

[illegible]





## A COMPUTER PROGRAM TO DRAW THE RESPONSE SURFACES

```

INTEGER I,J,NP
DIMENSION ZARR(48,48),XX(144),YY(144),ZZ(144)
DIMENSION WW(5184),WXL(4),WYL(4),WXH(4),WYH(4)
DATA M/48/,N/48/,NW/5184/,XL/1.0/,YL/1.0/,XH/12.0/,YH/12.0/,
+   IFR/1/,HTRAT/0.5/,XIU/0.0/,YIU/0.0/,ZIU/0.0/,NP/144/
DATA WXL(1),WXL(2),WXL(3),WXL(4)/0.0,120.0,0.0,120.0/
DATA WXH(1),WXH(2),WXH(3),WXH(4)/120.0,240.0,120.0,240.0/
DATA WYL(1),WYL(2),WYL(3),WYL(4)/120.0,120.0,0.0,0.0/
DATA WYH(1),WYH(2),WYH(3),WYH(4)/240.0,240.0,120.0,120.0/
DATA (XX(I),I=1,144)/1,1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2,2,
+2,
+2,3,3,3,3,3,3,3,3,3,3,3,3,4,4,4,4,4,4,4,4,4,4,4,5,5,5,5,5,5,5,5,
+5,5,5,5,6,6,6,6,6,6,6,6,6,6,6,6,7,7,7,7,7,7,7,7,7,7,7,7,8,8,8,8,
+8,8,8,8,8,8,9,9,9,9,9,9,9,9,9,9,9,9,10,10,10,10,10,10,10,10,10,
+10,10,10,11,11,11,11,11,11,11,11,11,11,11,11,12,12,12,12,12,12,12,
+12,12,12,12,12/
DATA (YY(I),I=1,144)/12,11,10,9,8,7,6,5,4,3,2,1,12,11,10,9,8,7,6,
+5,
+4,3,2,1,12,11,10,9,8,7,6,5,4,3,2,1,12,11,10,9,8,7,6,5,4,3,2,1,12,
+11,10,9,8,7,6,5,4,3,2,1,12,11,10,9,8,7,6,5,4,3,2,1,12,11,10,9,8,7,
+6,5,4,3,2,1,12,11,10,9,8,7,6,5,4,3,2,1,12,11,10,9,8,7,6,5,4,3,2,1,
+12,11,10,9,8,7,6,5,4,3,2,1,12,11,10,9,8,7,6,5,4,3,2,1,12,11,10,9,
+8,7,6,5,4,3,2,1/
DATA (ZZ(I),I=1,144)/123,112,102,75,87,75,132,127,102,107,120,87,
+115,123,127,152,132,122,153,132,132,133,127,123,
+165,142,105,107,112,138,145,160,113,127,145,128,
+112,128,160,132,132,138,135,160,132,82,133,133,
+88,102,125,152,130,97,77,133,158,152,122,108,
+130,102,107,132,137,127,135,107,113,157,113,103,
+145,122,142,127,137,145,148,145,145,130,112,115,
+142,107,113,92,145,127,147,145,115,123,132,140,
+130,120,157,127,137,137,152,165,113,125,178,148,
+135,172,207,165,197,138,142,160,125,183,163,153,
+142,162,145,133,160,168,143,113,202,205,230,105,
+167,148,235,240,212,227,192,118,158,243,187,168/
CALL OPEN
CALL RANGRD(NP,XX,YY,ZZ,M,XL,XH,N,YL,YH,ZARR,NW,WW)
CALL ISOFRA(IFR)
CALL HEIRAT(HTRAT)
CALL ISOSCA(XIU,YIU,ZIU)
DO 5 J=1,4
CALL WINDOZ(WXL(J),WXH(J),WYL(J),WYH(J))
CALL ISOPRJ(M,XL,XH,N,YL,YH,ZARR,J-1,NW,WW)
CONTINUE
CALL DEVEND
STOP
END

```

APPENDIX A6

POINT VELOCITY MEASUREMENTS ABOVE PACKED BEDS  
SUPPLIED BY MODIFIED GAS DISTRIBUTORS

Tables A6.1 and A6.2:      Modification A, P.H. = 6.12 cm

Tables A6.3 and A6.4:      Modification B with six baffles  
P.H. = 6.12 cm

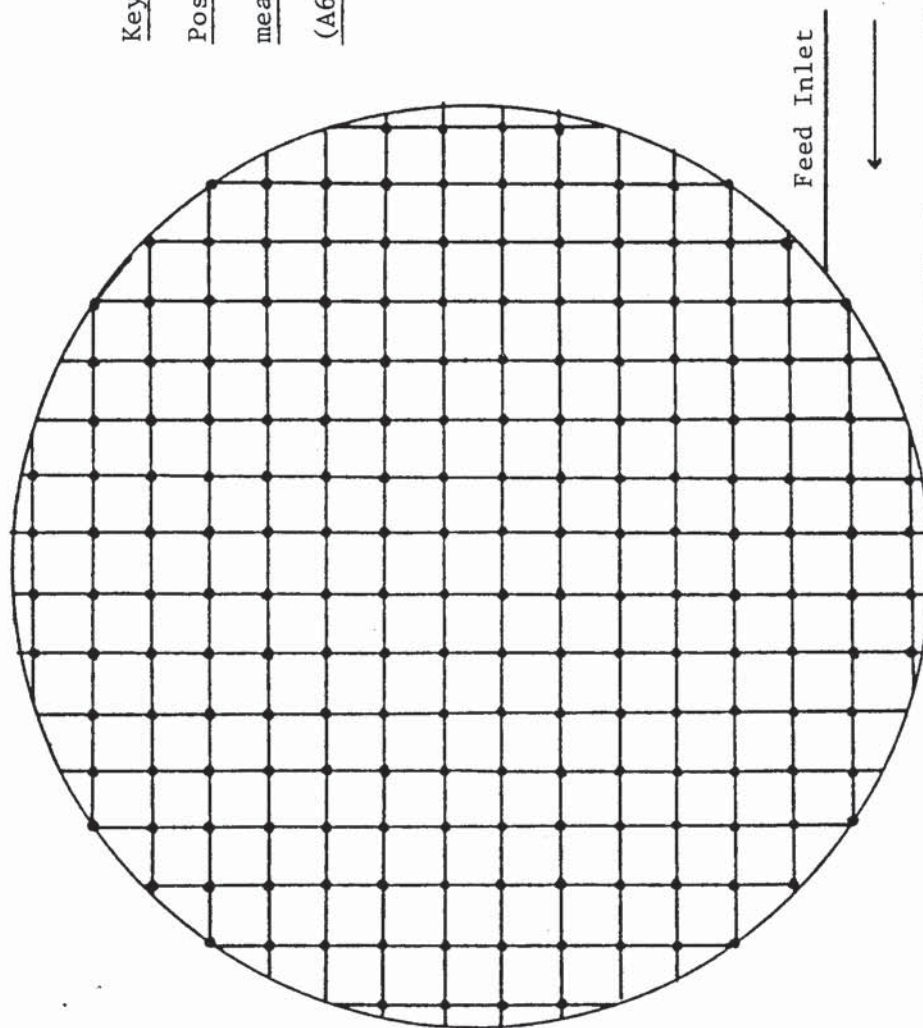
Tables A6.5 and A6.6:      Modification B with 24 baffles  
P.H.= 6.12 cm

Tables A6.7 and A6.8:      Modification C, P.H. = 6.12 cm

Tables A6.9 and A6.10:      Modification C + draw-off tray,  
P.H. = 6.12 cm.

Key Figure A6.1

Positions of the 200 point velocity  
measurements for tables (A6.1) to  
(A6.10)





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Table A6.1 Point Velocity Measurements (ft/min); Modification A, P.H. = 6 cm. P.S = 16 mm

Re = 60000

$\phi$  = 42.7

Table A6.2 Point Velocity Measurements (ft/min); Modification A, P.H. = 12 cm, P.S. = 16 mm

$$Re = 60000$$
$$\phi = 31.3$$



[illegible]

Table A6.3 Point Velocity Measurements (ft/min); Modification B with six baffles, P.H. = 6 cm, P.S. = 16 mm

$$Re = 60000$$
$$\phi = 33.5$$







160	95	120	195	160	155	135	155	130	135	130	140	165	165	130
195	70	115	140	105	55	55	205	125	105	70	125	80	125	145
110	215	20	110	125	100	125	125	70	130	30	100	140	105	90
145	220	130	165	90	135	120	120	105	90	165	105	125	165	110
	230	150	140	170	150	130	130	150	140	105	110	225	70	110
	160	90	120	210	100	110	110	125	35	170	135	160	105	110
	140	125	30	90	125	155	155	90	45	130	70	60	140	85
		105	180	140	145	150	150	110	180	100	90	130	240	190
			205	200	190	80	135	90	115	160	180	80		
						220	125	145	190					

Table A6.6 Point Velocity Measurements (ft/min); Modification B with 24 Baffles, P.H. = 12 cm, P.S. = 16 mm

Re = 60000

$\phi$  = 24.8

[illegible]
$$Re = 60000$$
$$\phi = 23.3$$



100

[illegible]

Table A6.9 Point Velocity Measurements (ft/min); Modification C + Draw-off Tray, P.H. = 6 cm, P.S. = 16 mm

$$Re = 60000$$
$$\phi = 22.8$$





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